BRAINWARE UNIVERSITY

ODD Semester Examinations 2021- 22

Programme – Master of Science in Mathematics - 2020 [M.Sc.(MATH)]

Course Name – Measure and Probability Theory

Course Code – MSCME302

(Semester III)

Time allotted : 1 Hour 15 Minutes

Full Marks : 60

(Multiple choise type question)

60 x 1 = 60

Choose the correct alternative from the following

(I) If the events $\{E_n\}$ are independent, then

A) $\sum_n P(E_n) = 1$

B) $\sum_n P(E_n) = \infty$

C) $\sum_n P(E_n) = 0$

D) Other

(II) Borel-Cantelli lemma is about

A) Sums of events

B) Sequences of events

C) Independence of events

D) Other

(III) If x_n has mean $\overline{x_n}$ and standard deviation σ_n then $x_n - \overline{x_n}$ converges to

A) 0 if $\sigma_n \to 0$

B) 1 if $\sigma_n \to 0$

C) 0

D) 1

(IV) If $X_n \to X$ and $Y_n \to Y$ in L^p then in L^p

A) $X_n + Y_n \to X + Y$

B) $X_n - Y_n \to X - Y$

C) All the above

D) Other

(V) Let f(x) is integrable on X. Then $F(E) = \int_E f(x)d\mu$ is

A) continuous but not absolutely continuous

B) absolutely continuous

C) bounded but not continuous

D) None of these

(VI) The sum of independent variables with binomial distributions

A) may not be a binomial distribution

B) must be a binomial distribution

C) never a binomial distribution

D) None of these

(VII) A card is drawn at random from a well-shuffled pack of cards. The probability that it is heart or queen is:

A) 4/13

B) 1/13

C) 5/13

D) 7/13

(VIII) Let f(x) be integrable on X. Then $F(E) = \int_E f(x)d\mu$ is defined for

A) finite subset of X only

B) all subset of X

C) measurable subset E of X

D) None of these

(IX) If a is a constant, then

A) $\int_E af(x)d\mu \le a\int_E f(x)d\mu$

B) $\int_E af(x)d\mu \ge a\int_E f(x)d\mu$

C) $\int_E af(x)d\mu = a\int_E f(x)d\mu$

D) $\int_E af(x)d\mu = |a| \int_E f(x)d\mu$

(X) If $X_n \Rightarrow X$ and X_n are uniformly integrable, then X is

A) always integrable
B) never uniformly integrable
C) may not be integrable
D) Other

(XI) Let θ be a irrational number then $\theta, 2\theta, 3\theta, \ldots$ is

A) Uniformly distributed modulo 1
B) Uniformly distributed module 0
C) Not uniformly distributed
D) Other

(XII) The distribution function of the Normal distribution given by the formula:

A) $\int_X \frac{1}{\sqrt{\sigma\pi}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}} d\mu$
B) $\int_X \frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}} d\mu$
C) $\int_X \frac{1}{\sqrt{\sigma\pi}} e^{\frac{(x-\bar{x})^2}{2\sigma^2}} d\mu$
D) $\int_X \frac{1}{2\sqrt{\sigma\pi}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}} d\mu$

(XIII) A sequence of random variables $\{x_n\}$ converges in probability to a random variable x. Then $\lim_{n\to\infty} p(|x_n - x| > 1/2)$ equals to

A) 0
B) 1/2
C) 1
D) Other

(XIV) A measure is said to be complete if every subset of a measurable set of ………. is also measurable.

A) finite measure
B) infinite measure
C) zero measure
D) None of these

(XV) The sum of independent variables with Poisson distributions c_1, c_2 is Poisson and has parameter …..

A) $c_1 . c_2$
B) $(c_1 .+ c_2)^2$
C) c_1+c_2
D) None of these

(XVI) Let μ is defined on an algebra of sets. Then μ is countably additive if for any finite (or countably infinite) system of (disjoint) sets X_n, we have

A) $\mu(\sum X_n) \geq \sum \mu(X_n)$
B) $\mu(\sum X_n) = \sum \mu(X_n)$
C) $\mu(\sum X_n) \leq \sum \mu(X_n)$
D) Other

(XVII) $\liminf E_n$

A) $\limsup E_n$
B) $\limsup E_n{}^c$
C) $(\limsup E_n)^c$
D) $(\limsup E_n{}^c)^c$

(XVIII) Let $y_k, k = 1,2,..$ be independent random variables with same distribution function and same finite mean m. Let $x_n = \frac{1}{n}\sum_{k=1}^{n} y_k$. Then

A) $x_n \to m$ in mean
B) $x_n \to m$ in probability
C) y_k have finite standard deviation
D) None of these

(XIX) If $X_n \to X$ and $X_n \to Y$ in probability then

A) $X = Y$
B) $X = Y$ a. e
C) $X \neq Y$ a. e
D) None of these

(XX) Let $\psi_n(t)$ has K-L representation $(a_n.G_n)$ and $\psi_n(t) \to \psi(t), \forall t$. Then $\psi(t)$

A) Must have K-L representation
B) May have K-L representation
C) Never have K-L representation
D) Other

(XXI) A sequence of random variables $\{X_n\}$ is convergent a. e. to 0 if for every $\varepsilon > 0$

A) $P\{|X_n| < \varepsilon, i.o\} = 0$
B) $P\{|X_n| > \varepsilon, i.o\} = 0$
C) $P\{|X_n| < \varepsilon, i.o\} = 1$
D) Other

(XXII) Which of the following statement is true?

A) Poisson distribution is a limiting case of binomial distribution
B) Binomial distribution is a limiting case of Poisson distribution

C) Binomial distribution is a limiting case of singular distribution

D) Singular distribution is a limiting case of binomial distribution

(XXIII) A sequence of random variables $\{X_n\}$ is convergent a. e. to X implies

A) $\lim P\{|X_n - X| \geq 1/2\} = 0$

B) $\lim P\{|X_n - X| \leq 1/2\} = 0$

C) $\lim P\{|X_n - X| > 1/2\} = 0$

D) $\lim P\{|X_n - X| < 1/2\} = 0$

(XXIV) The mean of the real-valued function $\alpha(x)$ of the random variable x is

A) $\int_X \alpha(x) d\mu$

B) $x \int_X \alpha(x) d\mu$

C) $\int_X x\alpha(x) d\mu$

D) Other

(XXV) $\mu_n \Rightarrow \mu$ implies $\int f d\mu_n \to \int f d\mu$ for every

A) Bounded, continuous real functions

B) Bounded real functions

C) Continuous real functions

D) Other

(XXVI) The characteristic function of the normal distribution is

A) $e^{-t^2/2}$

B) $e^{-t^2\sigma^2/2}$

C) $e^{-t^2/2\sigma^2}$

D) $e^{-\sigma^2/2}$

(XXVII) The upper limit, limsup E_n of a sequence $\{E_n\}$ of sets is the set of points which belong to E_n for ……… n.

A) infinitely many

B) finitely many

C) all

D) all but finitely many

(XXVIII) If $x_1, x_2, ..., x_n$ are independent random variables with standard deviations $\sigma_1, \sigma_2, ..., \sigma_n$ respectively. Then the standard deviation σ of their sum $x_1 + x_2 + ... + x_n$ satisfies the formula $\sigma^2 = \sigma_1^2 + \sigma_2^2 + .. + \sigma_n^2$ if n is

A) an odd

B) any positive integer

C) an even

D) a prime

(XXIX) If $f(x) \geq 0$ and $\int_E f(x) d\mu = 0$. Then

A) $f(x) = 0$ in E

B) $f(x) = 0$ a. e. in E

C) $f(x) > 0$ in E

D) $f(x) > 0$ a. e. in E

(XXX) $X_n \to 0$ in probability

A) if $E\left(\frac{|X_n|}{1+|X_n|}\right) \to 0$

B) if and only if $E\left(\frac{|X_n|}{1+|X_n|}\right) \to 0$

C) only if $E\left(\frac{|X_n|}{1+|X_n|}\right) \to 0$

D) Other

(XXXI) Convergence in L^3 implies that in

A) L^4

B) L^2

C) both in L^2 and L^4

D) None of these

(XXXII) $\mu_n(A) \to \mu(A), \forall A$, where A is a μ-continuity set if and only if $\int f d\mu_n \to \int f d\mu$, where f

A) bounded real functions

B) continuous real functions

C) bounded and continuous real functions

D) None of these

(XXXIII) Let F be a distribution function of a random variable. Then the number of discontinuity of F is

A) finite

B) countably infinite

C) at most countable

D) uncountable

(XXXIV) For a sequence of sets $\{X_n\}$, $\limsup X_n$ equals to

A) $\bigcup_{n=1}^{\infty} X_n$

B) $\bigcap_{n=1}^{\infty} X_n$

C) $\bigcup_{m=1}^{\infty}\bigcap_{n=m}^{\infty} X_n$

D) $\bigcap_{m=1}^{\infty}\bigcup_{n=m}^{\infty} X_n$

(XXXV) Which of the following set is not a Borel set of **R** of the set of all real numbers?

A) open set

B) closed set

C) half open interval

D) Other

(XXXVI) Which of the following statement is false for a distribution function?

A) It is decreasing

B) It is nondecreasing

C) It is right-continuous

D) It is discontinuous atmost at coutably many points

(XXXVII) A sequence of random variables $\{X_n\}$ is convergent a. e. to X. Then for every $\varepsilon > 0$

A) $\lim_{m\to\infty} P\{|X_n - X_{n'}| < \varepsilon, n' > n \geq m\} = 1$

B) $\lim_{m\to\infty} P\{|X_n - X_{n'}| < \varepsilon, n' > n \geq m\} = 0$

C) $\lim_{m\to\infty} P\{|X_n - X_{n'}| \geq \varepsilon, n' > n \geq m\} = 1$

D) $\lim_{m\to\infty} P\{|X_n - X_{n'}| > \varepsilon, n' > n \geq m\} = 0$

(XXXVIII) The characteristic function $\varphi(t)$ of a distribution function $F(x)$ satisfies

A) uniformly continuous

B) absolutely continuous

C) continuous but not uniformly

D) Other

(XXXIX) Let f(x) is integrable on X and $F(E) = \int_E f(x)d\mu$. Then $E_n \uparrow E$ implies

A) $F(E_n) = F(E), \forall n$

B) $F(E_n) \downarrow F(E)$

C) $F(E_n) \uparrow F(E)$

D) Other

(XL) The lower limit, $\liminf X_n$ of an decreasing sequence $\{X_n\}$ of sets equals to

A) $\bigcap_{n=1}^{\infty} X_n$

B) $\bigcup_{n=1}^{\infty} X_n$

C) X_1

D) Other

(XLI) The characteristic function of the binomial distribution $p_r = \binom{n}{r} p^r q^{n-r}$ is

A) $(p + qe^{it})^n$

B) $(q + pe^{it})^n$

C) $(p - qe^{it})^n$

D) $(p + qe^{-it})^n$

(XLII) The length of the interval (0, 1)U(0.5, 3)U(2.4, 4) is

A) 1

B) 2

C) 3

D) 4

(XLIII) Let $\psi(t)$ be a K-L function with representation (a, G). Then G is decreasing only in

A) the set of all positive real numbers

B) the set of all real numbers

C) an empty set

D) Other

(XLIV) If $X_n \Rightarrow X$ and $X_n - Y_n \Rightarrow 0$. Then

A) $X_n Y_n \Rightarrow X$

B) $X_n Y_n \Rightarrow 2X$

C) $Y_n \Rightarrow X$

D) Other

(XLV) The area of the rectangle $[2,3]\times[5,7]$ is

A) 12

B) 3

C) 2

D) 1

(XLVI) If $x_n \to x$ in mean then

A) $x_n \to x$ in probability

B) $F_n(x) \to F(x)$

C) All the above

D) None of the above

(XLVII) Let $\alpha(x_1, x_2)$ be a function of two independent variable x_1, x_2 and $X = X_1 \times X_2$. Then

A) $\int_X \alpha(x_1, x_2) d\mu = \iint_X \alpha(x_1, x_2) d\mu^2$
B) $\int_X \alpha(x_1, x_2) d\mu = \iint_X \alpha(x_1, x_2) d\mu_1 d\mu_2$
C) $\int_X \alpha(x_1, x_2) d\mu = 0$
D) Other

(XLVIII) If $x_n \to x$ in probability then

A) $F_n(x_n) \to F(x)$
B) $F_n(x) \to F(x)$
C) $F(x_n) \to F(x)$
D) Other

(XLIX) If x_1, x_2, x_3 are three independent random variables with standard deviations $\sigma_1, \sigma_2, \sigma_3$ respectively. Then the standard deviation σ of their sum $x_1 + x_2 + x_3$ satisfies the formula:

A) $\sigma^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2$
B) $\sigma^2 \le \sigma_1^2 + \sigma_2^2 + \sigma_3^2$
C) $\sigma^2 \ge \sigma_1^2 + \sigma_2^2 + \sigma_3^2$
D) Other

(L) If the events $\{E_n\}$ are pair-wise independent, then the value of $P(\limsup E_n) = 0$ if

A) $\sum_n P(E_n) = \infty$
B) $\sum_n P(E_n) < \infty$
C) $\sum_n P(E_n) = 0$
D) Other

(LI) The sum of independent variables with Poisson distributions

A) . must be a Poisson distribution
B) may not be a Poisson distribution
C) never be a Poisson distribution
D) Other

(LII) Consider the finite number of disjoint open intervals I_k, and $I = \cup I_k$, then the length of I, that is, $l(I)$ is

A) $\le \sum l(I_k)$
B) $\ge \sum l(I_k)$
C) $= \sum l(I_k)$
D) 0

(LIII) Find the volume of $[2,3] \times [5,7] \times [8,12]$

A) 6
B) 8
C) 10
D) Other

(LIV) The sum of two independent variables has normal distributions then

A) At-least one has normal distribution
B) Both variables have normal distribution
C) c. At-most one variable has normal distribution
D) Both variables may not have normal distribution

(LV) An additive function is countably additive if it is

A) only continuous from below
B) only finite and continuous at 0
C) either continuous from below or finite and continuous at 0
D) Other

(LVI) A measure μ on an algebra **R** of subsets of a set X is called a probability measure if

A) μ(X)=1
B) μ(X)=0
C) μ(X) is finite
D) μ(X) is infinity

(LVII) The mean of a Poisson distribution with parameter 10 is

A) 10
B) -10
C) 100
D) -100

(LVIII) For every sequence $\{F_n\}$ of distribution functions there exists a subsequence $\{F_{nk}\}$ and a non-decreasing, ……….. function F such that $\lim F_{n_k}(x) = F(x)$ at continuity points x of F.

A) left-continuous
B) right-continuous
C) continuous
D) Other

(LIX) Let $\psi(t)$ be a K-L function with representation (a, G). Then G is…..in R

A) Non-decreasing
B) Non-increasing
C) Strictly increasing
D) Strictly decreasing

(LX) A $K-L$ function is…… in every finite interval.

A) unbounded

B) bounded

C) continuous

D) Other