BRAINWARE UNIVERSITY

Term End Examination 2018 - 19

Programme –Bachelor of Computer Applications/Bachelor of Science (Honours) in Computer Science

Course Name -Discrete Structures

Course Code - BCA202/BCS202

(Semester – 2)

Time allotted: 3 Hours **Full Marks : 70**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group –A

(Multiple Choice Type Questions) 10 x 1 = 10

1. ***Choose the correct alternative from the following:***

(i) Contrapositive of "$\sim p \rightarrow q$" is

a. $p \rightarrow q$
b. $\sim q \rightarrow \sim p$
c. $\sim q \rightarrow p$
d. $q \rightarrow \sim p$

(ii) The truth value of the statement $x^2 = x$ holds for all real values of x is

a. T
b. F
c. Neither T nor F
d. none of these

(iii) In how many ways 7 different beads can be arranged to form a necklace?

a. 250
b. 300
c. 360
d. 350

(iv) The least number of people, 4 of whom will have same birthday of the week is,

a. 18
b. 42
c. 28
d. 22

(v) If $f(x) = x\sec(x)$, then $f(0) =$

a. -1
b. 0
c. 1
d. $\sqrt{2}$

(vi) If $f(x) = \tan^{-1}(x)$ and $g(x) = \tan(x)$, then $(gof)(x) =$

a. $\tan^{-1}x\tan(x)$
b. $\tan^{-1}x\cot(x)$
c. x
d. $\tan^{-1}x\sin(x)$

(vii) If a, b are elements of a group G, then $(ba)^{-1}$ =

a. $a^{-1} b^{-1}$ b. $b^{-1} a^{-1}$

c. $a^{-1} b$ d. $b^{-1} a$

(viii) G = { 1, -1, i, - i } is a group under multiplication. Then the inverse of - i is

a. 1 b. -1

c. i d. -i

(ix) If a graph has 6 vertices and 15 edges, then the size of its adjacency matrix is

a. 6X6 b. 6X15

c. 15X6 d. 15X15

(x) Chromatic number of a complete graph with 15 number of vertices is

a. 12 b. 13

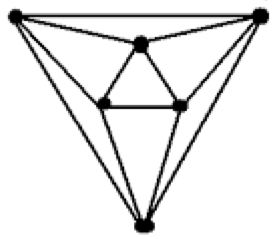
c. 14 d. 15

Group – B

(Short Answer Type Questions) 3 x 5 = 15

Answer any three from the following :

2. Show that $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology. [5]
3. Find the number of integers between 1 and 500 which are multiples of 3,5 or 7 . [5]
4. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 3x + 1,\ x \in \mathbb{R}$. Examine if f is bijective. [5]
5. Let $S = \{1, \omega, \omega^2\}$ where $\omega^3 = 1$. Prove that S is an abelian group with respect to multiplication. [5]
6. Draw the dual of the following graph : [5]



Group – C

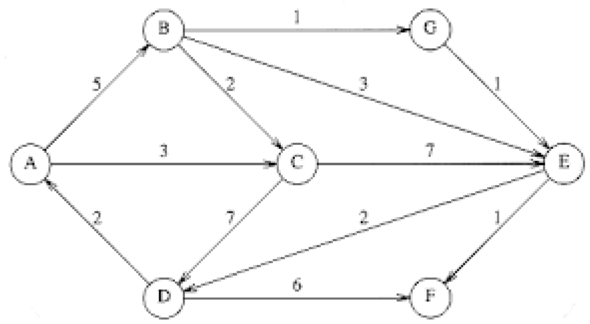
(Long Answer Type Questions) 3 x 15 = 45

Answer any three from the following :

7. (a) Let us consider the set of all 2x2 real matrices $\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : \det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = 1\right\}$. [7]

Prove that H is a subgroup of $GL(2,\mathbb{R})$.

(b) Apply Dijkstra's method to find the shortest path and distance between the two vertices A & E in the given Digraph. [8]



8. (a) Find the minimum number n of integers to be selected from S={1,2,…….,9} so that [2+3]
   i. The sum of two of the n integers is even.
   ii. The difference of two of the n integers is 5.

(b) State and proof De Morgan's laws. [5]

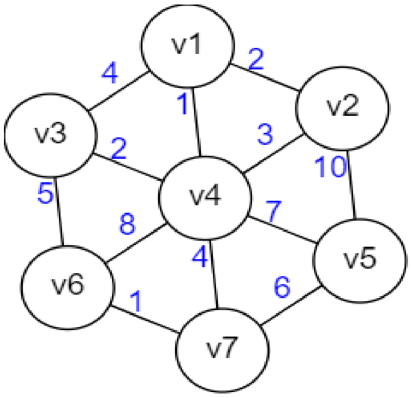
(c) A relation ρ on the set $\mathbb{N}$ is given by $\rho = \{(a,b) \in \mathbb{N}\times\mathbb{N}:\ a|b\}$. Examine if ρ is an equivalence relation. [5]

9. (a) Prove that Chromatic polynomial for a complete graph with n vertices is x(x-1)(x-2)……..(x-n+1). [6]

(b) 7 boys and 5 girls are to be seated in a row . In how many ways can they be seated if [3+3+3]
   (i) all boys are to be seated together and all girls are to be seated together.
   (ii) no two girls should be seated together.
   (iii) the boys should occupy extreme positions .

10. (a) Let $f:\mathbb{R}\to\mathbb{Z}$ be defined by $f(x)=[x], x\in\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{Z}$ be defined by $g(x)=x+\frac{1}{2}, x\in\mathbb{Z}$. Examine whether f and g are invertible. [6]

(b) Apply Kruskal's algorithm to find a shortest spanning tree of the following graph : [7]



(c) Prove that for a 'p-regular' graph with n number of vertices, the number of edges should be exactly $\frac{np}{2}$. [2]

11. (a) Prove that for two sets A and B, [3]
$n(A\cup B)=n(A)+n(B)-n(A\cap B)$.

(b) In a group (G, $*$), prove that, [3+3]

i) $a*b=a*c$ implies $b = c$.

ii) $b*a=c*a$ implies $b = c$.

(c) Prove that a simple graph with n number of vertices and k number of components can have maximum $\frac{(n-k)(n-k+1)}{2}$ number of edges. [6]
