BRAINWARE UNIVERSITY

Term End Examination 2018 - 19

Programme –Bachelor of Computer Applications

Course Name -Algebra and Calculus

Course Code - BCA203C

(Semester – 2)

Time allotted: 3 Hours **Full Marks : 70**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group –A

(Multiple Choice Type Questions) 10 x 1 = 10

1. ***Choose the correct alternative from the following:***

(i) If $f(x,y) = x + y$ then df =

a. dx b. dy

c. $dx + dy$ d. None of these

(ii) Every integer n is relatively prime to

a. 1 b. 2

c. 3 d. 4

(iii) For a given graph G having v vertices and e edges that is connected and has no cycles, which of the following statements is true?

a. v=e b. v = e+1

c. v + 1 = e d. None of these

(iv) For which of the following combinations of the degrees of vertices would the connected graph be Eulerian?

a. 1,2,3 b. 2,3,4

c. 3,4,5 d. 1,3,5

(v) If $f(x,y) = x^2 + y^2$, then $f_{xy}(x,y) =$

a. 1 b. 0

c. 2 d. x+y

(vi) If $u = \log(x^2 + y^2)$, then $u_{xx} + u_{yy} =$

a. 0 b. $\frac{x}{y}$

c. $\frac{y}{x}$ d. 1

(vii) $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2} =$

a. 0 b. 1

c. $\frac{1}{2}$ d. None of these

(viii) $\int_1^3 \int_2^{-1} \int_1^2 dxdydz =$

a. -6 b. 3

c. -3 d. 2

(ix) If the roots of linear second order differential equation are real and distinct,then the general solution will contains

a. two constants and two exponentials b. two constants and one exponentials

c. sinusoidal functions and exponentials d. one constant and two exponentials

(x) $\int_0^2 \int_0^{\frac{\pi}{2}} \cos x dxdy =$

a. 2 b. -1

c. sin2 d. None of these

Group – B

(Short Answer Type Questions) 3 x 5 = 15

Answer any three from the following:

2. Show that if a|b and a|c then a|(bx+cy),where a,b,c,x,y are integers. [5]
3. Prove by mathematical induction that $n^3 + 2n$ is divisible by 3 $, n \in N$. [5]
4. Prove that for a complete graph with n number of vertices the number of edges is exactly $\frac{n(n-1)}{2}$. [5]
5. If $u = f(y-z, z-x, x-y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. [5]
6. Evaluate $\iint_R (x^2 + y^2) dxdy$ over the region enclosed by the triangle having vertices at (0,0) ,(1,0) and (1,1). [5]

Group – C

(Long Answer Type Questions) 3 x 15 = 45

Answer any *three* from the following

7. (a) Evaluate $\iiint (x+y+z+1)^4\,dxdydz$ over the three dimensional region: $x \geq 0$, $y \geq 0$, $z \geq 0$, $x+y+z \leq 1$. [6]

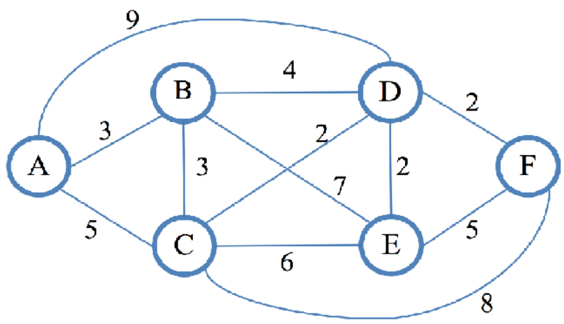
   (b) Solve: $p^3 - p\left(x^2+xy+y^2\right)+x^2y+xy^2=0$, Where P = $\frac{dy}{dx}$ [4]

   (c) Find the maxima and minima of the function $x^3+y^3-3x-12y+20$. Find also the saddle point. [5]

8. (a) For the function, $f(x,y)=\begin{cases}\dfrac{x^3+y^3}{x-y}, & x \neq y\\ 0 & ,x=y\end{cases}$ [7]

   prove that f(x, y) is not continuous at (0,0) but $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ exist at (0,0).

   (b) Apply Dijkstra's method to find the shortest path and shortest distance between the two vertices A & F in the given graph: [8]



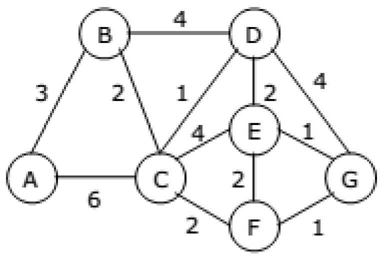
9. (a) Solve: $4x \equiv 3 (\text{mod } 7)$ [4]

   (b) Using double integral, find the area of the triangle whose vertices are (1,3), (0,0) and (1,0). [5]

   (c) Solve by the method of variation of parameters: $\frac{d^2y}{dx^2}+9y=\sec 3x$ [6]

10. (a) A fruit seller orders mangoes and oranges for Rs.1000.If one basket of mangoes costs Rs 20 and one basket of oranges cost Rs.172, how many baskets of each type does he order? [6]

(b) Apply Kruskal's algorithm to find the shortest spanning tree of the following graph [6]



(c) If $v = \sin^2 u$ where $u = x^3 y^4$, find $\dfrac{\partial v}{\partial y}$. [3]

11. (a) Prove that if $a \equiv b(\text{mod } m)$, then $ac \equiv bc(\text{mod } m)$ but if $ac \equiv bc(\text{mod } m)$ then $a \equiv b\left(\text{mod}\dfrac{m}{\gcd(c,m)}\right)$ [5]

(b) Solve $x^2D^2y - 3xDy + 5y = x^2 \sin\log x$, where D= $\frac{d}{dx}$ [5]

(c) Draw a graph with four edges and four vertices having degrees 2,2,3,3 . If it is not possible,find the reason. [2]

(d) Examine if it feasible to draw a graph with seven vertices having degrees 3,5,2,7,4,6,8. If not, find the reason. [3]
