BRAINWARE UNIVERSITY

Coursework Examination 2018 – 19 (June 2019)

Programme – Doctorate of Philosophy in Mathematics

Course Name – Functional Analysis

Course Code – PHD-MAT-01

Time allotted: 4 Hours **Full Marks : 100**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group –A

(Objective Type Questions) **10 x 1 = 10**

1. ***Answer the following*** **:**

(i) The metric for the sequence space l^∞ is defined as $d(x,y) = \sup_{i \in N} |x_i + y_i|$. **(*True/False*)**

(ii) Every compact metric space is separable. **(*True/ False*)**

(iii) l^p is a Banach space if $1 \le p < \infty$. **(*True / False*)**

(iv) The dual space of R^n is not R^n. **(*True / False*)**

(v) Define normal operator.

(vi) The space $l^p, p \neq 2$ is an inner product space. **(*True/ False*)**

(vii) Write the Bessel's inequality.

(viii) Completion of normed linear space is called …………..

(ix) An element x of an inner product space X is said to be orthogonal to an element $y \in X$ if …………

(x) State Zorn's Lemma.

Group – B

(Short Answer Type Questions) **6 x 5 = 30**

Answer any *six* from the following:

2. Show that $L_p(0,1)$ is a metric space ($p \geq 1$). 5

3. State and prove Holder's inequality for integrals. 5

4. If a normed space X has the property that the closed unit ball $M = \{x \mid \|x\| \leq 1\}$ is compact, then X is finite dimensional. Verify it. 5

5. Construct an example of a normed linear space which is not a Banach space. 5

6. Prove the Schwarz inequality. When does the inequality become equality? Explain. 5

7 Define self-adjoint operator. Show that a continuous linear operator $A: H \to H$ is self-adjoint iff (Ax, x) is real for all x. 5

8. State the Hahn Banach Theorem for real and complex vector spaces. Prove the theorem for real vector space. 5

9. State and prove the Uniform Boundedness Theorem. 5

Group – C

(Long Answer Type Questions) **6 x 10 = 60**

Answer any *six* from the following:

10. Show that the Euclidean space R^n is a complete metric space. 10

11. Let $X = (X, d)$ be a metric space. Then show that : 10

    (a) A convergence sequence in X is bounded and its limit is unique.

    (b) If $x_n \to x$ and $y_n \to y$ in X, then $d(x_n, y_n) \to d(x, y)$.

12. Prove that the dual space of l^1 is l^∞. 10

13. State and prove F. Riesz's Lemma. 10

14. In completeness theorem of bounded linear operator, if Y is a Banach space, then prove B(X,Y) is also a Banach space. 10

15. State and prove the Pythagorean Theorem and Projection Theorem in a Hilbert space. 10

16. State the Baire's Category Theorem. Justify the statement with proof. Is the converse also true? 10

17. State and prove the Open Mapping Theorem. 10
