BRAINWARE UNIVERSITY

Course – BSc(CS)

Discrete Mathematics (BCSC 102)

(Semester – 1)

Time allotted: 3 Hours   **Full Marks : 70**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group –A

(Multiple Choice Type Question)   1 x 10 = 10

1. Choose the correct alternative for the following : (*Any Ten*)

(i) A _____ is an ordered collection of objects.

  (a) Relation   (b) Function

  (c) Set   (d) Proposition

(ii) Power set of empty set has exactly _____ subset.

  (a) 1   (b) 2

  (c) 3   (d) 0

(iii) What is the Cartesian product of A = {1, 2} and B = {a, b}?

  (a) {(1, a), (1, b), (2, a), (b, b)}   (b) {(1, 1), (2, 2), (a, a), (b, b)}

  (c) {(1, a), (2, a), (1, b), (2, b)}   (d) {(1, 1), (a, a), (2, a), (1, b)}

(iv) What is the Cardinality of the Power set of the set {0, 1, 2}?

  (a) 8   (b) 6

  (c) 7   (d) 9

(v) A graph G is called a ..... if it is a connected acyclic graph

(a) Cyclic graph   (b) Regular graph

(c) Tree   (d) Null graph

(vi) In an undirected graph the number of nodes with odd degree must be

(a) Zero   (b) Odd

(c) Prime   (d) Even

(vii) Which of the following two sets are disjoint?

(a) {1, 3, 5} and {1, 3, 6}   (b) {1, 2, 3} and {1, 2, 3}

(c) {1, 3, 5} and {2, 3, 4}   (d) {1, 3, 5} and {2, 4, 6}

(viii) The complement of the set A is _____.

(a) A – B   (b) U – A

(c) A – U   (d) B – A

(ix) If a set contains exactly m distinct elements where m denotes some non negative integer then the set is

(a) Finite   (b) Infinite

(c) Semi-finite   (d) None of these

(x) Let f and g be the functions defined by f(x)= 2x+3 & g(x)= 3x+2 then composition of f and g is

(a) 6x+6   (b) 5x+5

(c) 6x+7   (d) 5x+6

(xi) Which of the following is not a type of graph?

(a) Euler   (b) Hamiltonian

(c) Tree   (d) Path

Group – B

(Short Answer Type Question)   3 x 5 = 15

Answer any *three* from the following:

2. What do you mean by POSET? How it differs from Lattice?

3. Differentiate between Eulerian and Hamiltonian Graph with suitable example.

4. What do you mean by the term Connected and Disconnected Graph? How they differ?

5. There are exactly three types of students in a school: the geeks, the wannabees, and the athletes. Each student is classified into at least one of these categories. And the total number of students in the school is 1000. Suppose that the following is given:

The total number of students who are geeks is 310.
The total number of students who are wannabees is 650.
The total number of students who are athletes is 440.
The total number of students who are both geeks and wannabees is 170.
The total number of students who are both geeks and athletes is 150.
The total number of students who are both wannabees and athletes is 180.

What is the total number of students who fit into all 3 categories?

6. Find the product AB where A and B are matrices given by:

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 6 \\ 1 & 4 \\ 5 & 2 \end{bmatrix}$$

Group – C

(Long Answer Type Question)   3 x 15 = 45

Answer any *three* from the following:

7. (a) Show the recursive form of the following Recurrence Relation -

   $X_n = 2X_{n-1} + 1$ with initial condition $n_1 = 2$

   (b) Differentiate between Null Graph and Trivial Graph using an example. Can a Null Graph can be transformed into a Trivial Graph? If yes how if no why.

   (c) Differentiate between Bipartite and Complete Bipartite Graph.   (5 + 5 + 5)

8. (a) Test for consistency and solve the following system of equation using matrix method :

$x + y + z = 6$

$x - y + 2z = 5$

$3x + y + z = 8$

(b) Does there exist a simple graph with seven vertices having having degree sequences (1, 2, 2, 4, 5, 6, 6)? If no why if yes how.

(c) Construct the truth table for the following -

$(A \wedge B) \vee (\neg A \wedge B) \vee (A \wedge \neg B) \vee (\neg A \wedge \neg B)$ $(5 + 5 + 5)$

9. (a) Prove the following using mathematical induction:

$1+2+3+4+5+...+ n = \dfrac{n(n+1)}{2}$

(b) Let A and B be two finite sets such that n(A) = 20, n(B) = 28 and n(A ∪ B) = 36, find

n(A ∩ B).

(c) In a group of 100 persons, 72 people can speak English and 43 can speak French. How many can speak English only? How many can speak French only and how many can speak both English and French? $(7 + 5 + 3)$

10. (a) Prove the following using mathematical induction:

$1 + 3 + 5 + 7 + ... + 2n\text{-}1 = n^2$

(b) Among 18 students in a room, 7 study mathematics, 10 study science, and 10 study computer programming. Also, 3 study mathematics and science, 4 study mathematics and computer programming, and 5 study science and computer programming. We know that 1 student studies all three subjects. How many of these students study none of the three subjects?

(c) Construct the truth table for the following -

$(A \wedge B) \vee (\neg A \wedge B) \Leftrightarrow (A \wedge \neg B) \vee (\neg A \wedge \neg B)$ $(3 + 6 + 6)$

11. (a) Define the following terms -

i) Eccentricity of a Vertex

ii) Radius of a Connected Graph

iii) Diameter of a Graph

(b) Differentiate between Isolated and Pendant Vertex.

(c) Construct the truth table for the following –

$(\neg A \wedge (\neg B \wedge C)) \rightarrow (\neg A \wedge B) \vee (A \wedge \neg B) \vee (\neg A \wedge \neg B)$ $(7 + 3 + 5)$