BRAINWARE UNIVERSITY

Course – B.Sc.(CS)

Mathematics - I (BCS103)

(Semester –1)

Time allotted:3 Hours **Full Marks : 70**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group –A

(Multiple Choice Type Questions) 10 x 1 = 10

1. *Choose the correct alternative from the following*

(i) If A is a non-null square matrix then $A+A^T$ is a

a. Symmetric matrix
b. Skew-symmetric matrix
c. Null matrix
d. None of these

(ii) Trace of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$

a. 6
b. 5
c. 4
d. 7

(iii) The inverse of the matrix $\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$

a. $\begin{pmatrix} 2 & -3 \\ 4 & 6 \end{pmatrix}$
b. $\begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix}$
c. does not exit
d. $\begin{pmatrix} -2 & 4 \\ -3 & 6 \end{pmatrix}$

(iv) If A and B are sets and $A \cup B = A \cap B$, then

a. $A = \Phi$
b. $B = \Phi$
c. $A = B$
d. none of these

(v) The number of elements in the power set of the set {a, b} is

a. 2
b. 4
c. 6
d. 8

(vi) If p: “anil is rich” and q: “kanchan is poor” then the symbolic form of the statement “Either Anil or Kanchan is rich” is

a. $p \vee q$ b. p∨~q

c. $\sim p \vee q$ d. $\sim(p \vee q)$

(vii) For the statement p and q □ $(p \vee q)$ implies

a. ~ p ∧ ~ q b. ~p∨~q

c. $(p \vee q)$ d. None of these

(viii) The Fibonacci sequence is

a. 0,1,2,3,5,8……. b. 0,1,2,3,4,5,…..

c. 1,1,2,3,5,8,……. d. 0,-1,3,-6,10,…….

(ix) The solution of the recurrence relation $a_n = 2a_{n-1} + 1, \ with \ a_0 = 1$

a. 2^n b. 2^n-2

c. 2^n+1 d. 2^n-1

(x) The root of a binary tree is the vertex having degree

a. 1 b. 2

c. 3 d. 4

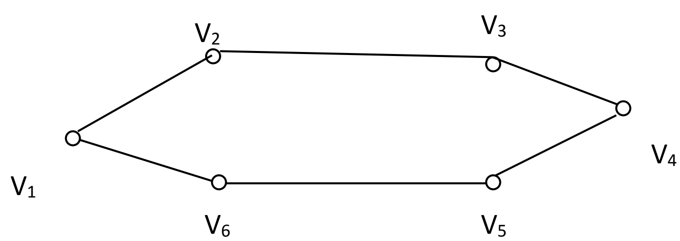
Group – B

(Short Answer Type Questions) 3 x 5 = 15

Answer any *three* from the following

2. (a) If the two propositions p and q are false then find whether the proposition (p ∨ q) ∧ (~p) ∨ (~q) is true or false.
(b) How many vertices are necessary to construct a graph with exactly 12 edges in which each vertex is of degree 3? [2+3]

3. (a)Define complement of a graph. Draw the complement of the following graph:



(b) Show that the mapping f is neither injective nor subjective

$$f : \mathrm{R} \rightarrow R \ \ defined \ by \ \ f(\mathrm{x}) = |x|, x \in R$$ [2 + 3]

(a)Find the generating function corresponding to the sequence {1, -1, 1, -1, 1, -1 …}

4. (b) Prove that every orthogonal matrix A is non-singular and det(A)= ±1 [2+3]

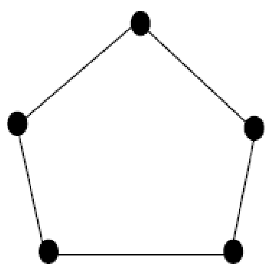
5. (a) Prove that the number of odd degree vertices in any graph is even.

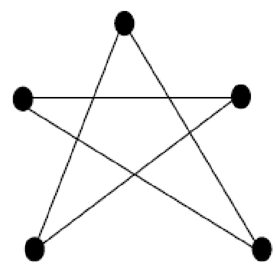
(b) Find if possible to form AB and BA ,stating with reasons where the operations do not hold where,

$A=\begin{bmatrix} 2 & 4 & -1 \\ 1 & -2 & 3 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 4 \\ -2 & 5 \\ -3 & 0 \end{bmatrix}$

[2+3]

6. Examine whether the following two graphs are isomorphic or not:





[5]

Group – C

(Long Answer Type Questions) 3x 15= 45

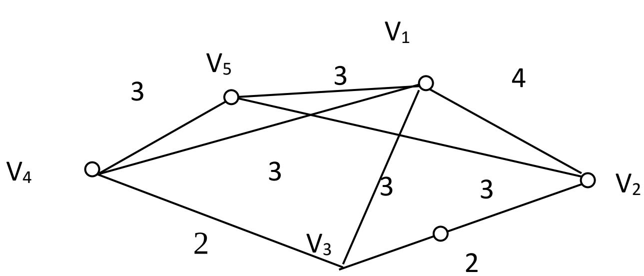
Answer any *three* from the following

7. (a) Find the inverse of the matrix $A=\begin{pmatrix} 2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{pmatrix}$ [5]

(b) Solve, by characteristic root method, the recurrence relation $a_n = 3a_{n-1} + 4a_{n-2}, a_0 = 0, a_1 = 5$ .Hence find a_8 [6]

(c) Show that number of vertices in the binary tree is always odd [4]

8. (a) Find by prim's algorithm a minimal spanning tree from the following graph

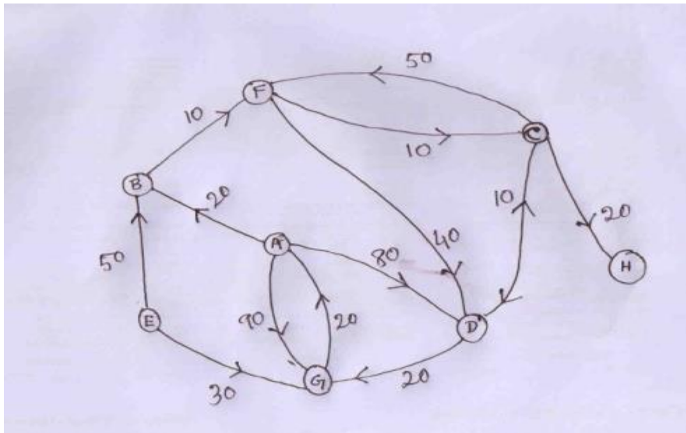


[6]

(b) Find the CNF(Conjunctive normal form) of the following statement
$\sim(p \vee q) \leftrightarrow (p \wedge q)$ [4]

(c) A graph G has a spanning tree if and only if G is connected [5]

9. (a) Applying Dijkstra's method to find the shortest path and distance between the two vertices **A** & **G** in the given Digraph.



[8]

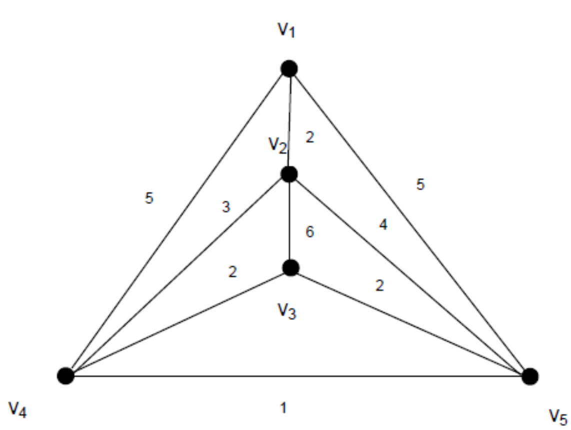
(b) Solve the recurrence relation
$a_n + 5a_{n-1} + 6a_{n-2} = 3n^2 - 2n + 1$ [7]

10. (a) Prove that, a simple graph with n number of vertices and k number of components can have maximum $\frac{(n-k)(n-k+1)}{2}$ number of edges. [6]

(b) Show by truth table that the following statement formula is a Tautology:
$\{(p \wedge \sim q) \rightarrow r\} \rightarrow \{p \rightarrow (q \vee r)\}$ [5]

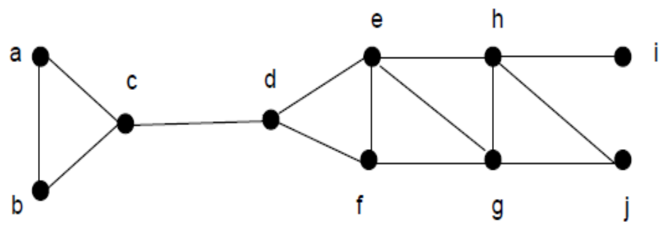
(c) Using generating function solve the recurrence relation
$a_n = a_{n-1} + n, \text{ with } a_0 = 1$ [4]

11. (a) Apply Kruskal's algorithm to find an optimal spanning tree for the given weighted graph



[5]
[4]

(b) Use DFS and BFS to construct a spanning tree for the following graph(Choose ‘a’ as the root):



(c) Prove that Chromatic polynomial for a complete graph with n vertices is x(x-1)(x-2)……..(x-n+1).

[6]