BRAINWARE UNIVERSITY

Course – BCA

Discrete Mathematics (BCAC102)

(Semester – 1)

Time allotted: 3 Hours   **Full Marks : 70**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group –A

(Multiple Choice Type Questions)   10 x 1 = 10

1. *Choose the correct alternative from the following*

(i)   The proposition $(p \wedge q)$ is equivalent to

   a. $(p \vee q)$ b. $p \wedge q$

   c. $p \vee q$ d. None of these

(ii)   If A, B and C are sets, then $A - (B \cup C)$ is equal to

   a. $(A - B) \cup (A - C)$ b. $(A - B) \cap (A - C)$

   c. $(A - B).(A - C)$ d. None of these

(iii) If n(A)=7, n(A∪B)=15 and n(A∩B)=3 then n(B) equals

   a. 21   b. 11

   c. 25   d. 19

(iv) A partial ordered relation is transitive, reflexive and

   a. Antisymmetric   b. Antireflexive

   c. Asymmetric   d. Irreflexive

(v)   An one-to-one function is known as

   a. Injective function   b. Surjective function

   c. Bijective function   d. None of these

(vi) What is the cardinality of the set of odd positive integers less than 10?

   a. 10   b. 5

   c. 3   d. None of these

(vii)   Identity matrix is always a

       a. Null matrix   b. Square matrix

       c. Triangular matrix   d. None of these

(viii)   Which is the correct statement about the function $f(x) = 2x$ if $f:\mathbb{Z} \rightarrow \mathbb{Z}$

       a. $f(x)$ is one-to-one and onto   b. $f(x)$ is not one-to-one but onto

       c. $f(x)$ is one-to-one but not onto   d. $f(x)$ is neither one-to-one nor onto

(ix)   If A is a non-null matrix then $(A^{-1})^{-1}$ is equals

       a. A^{-2} b. A

       c. A^{-1} d. None of these

(x)   If $A \subseteq B \ and \ B \subseteq C$ then $A \subseteq C$

       a. True   b. False

       c. Sometimes true   d. None of these

Group – B

<div align="center">(Short Answer Type Questions)   3 x 5 = 15</div>

Answer any *three* from the following

2.   Prove that $A - (B \cup C) = (A - B) \cap (A - C)$ [5]

3.   Define is symmetric and skew symmetric matrix? Give an example.   [5]

4.   Obtain the DNF of $\sim P \wedge (Q \leftrightarrow R)$ [5]

5.   If $g(x) = 9x + 6$ then find$g^{-1}(x)$. Hence show that $g^{-1}o(x) = x.$ [5]

6.   Verify the following relation defined on set of straight line L is

    equivalence or not.

$$R = \{(l_1, l_2): l_1 \, is \parallel l_2, l_1, l_2 \in L\}$$ [5]

Group – C

<div align="center">(Long Answer Type Questions)   3 x 15 = 45</div>

Answer any *three* from the following

7.   (a)   If $A = \{1,2,3\}, B = \{3,4,5\} \ and \ C = \{4,5,6\}$ then

        i) Prove that $(A \times B) - (A \times C) = A \times (B - C)$

        ii) Examine whether $(A \cap B) \times (B - A)$ is a subset of $(A \times B)$?

                                                                                 [4+4]

    (b)   Solve the following System of equation using matrix method:
            $4x - 3y + 2z = 11$
            $x + 2y + 4z = 16$
            $3x + 2y - z = 5$ [7]

8. (a) A small town has a total population 25000, out of which 13000 read 'The Statesman' and 10500 read 'The Hindustan Times' whereas 3500 read both papers. Find the percentage of population who read neither of these papers [6]

   (b) Prove that $[(p \to q) \land (q \to r)] \to (p \to r)$ is a tautology [7]

   (c) Define complete graph with example. [2]

9. (a) Solve the following recurrence relation:

   $f(n) - 2f(n-1) + f(n-2) = 0, \quad f(0) = 2, f(1) = 5$ [8]

   (b)
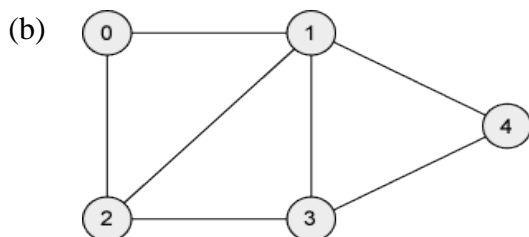
   Find the degree of each vertex of the given weighted graph. Hence show that sum of degrees of the vertices is twice the number of edge. [7]

10. (a) Express the following matrix as a sum of a symmetric and a skew-symmetric matrix.

   $$\begin{bmatrix} 5 & -3 & 6 \\ -2 & 7 & 3 \\ 0 & 1 & -2 \end{bmatrix}$$ [6]

   (b) Prove the following using mathematical induction:

   $1^2 + 2^2 + 3^2 + \cdots + n^2 = n(n+1)(2n+1)/6$ [7]

   (c) Define power set. Give an example. [2]

11. (a) If $f: R \to R$ is a mapping defined by
   $$f(x) = 3x + 5$$
   Verify whether f is a bijective mapping. [6]

   (b) What is transitive relation? Give an example. [3]

   (c) Show that the following pairs of propositions are logically equivalent

   $p \to (q \lor r) \equiv (p \to q) \lor (p \to r)$ [6]