BRAINWARE UNIVERSITY

Term End Examination 2018 - 19

Programme– B.Tech. CSE / B. Tech. ECE

Course Name - Calculus

Course Code –BMAT010101

(Semester – 1)

Time allotted:3 Hours **Full Marks: 70**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group –A

(Multiple Choice Type Questions) 10 x 1 = 10

1. ***Choose the correct alternative from the following :***

(i) For a function *f*(*x*) the expression

$$\frac{h^n(1-\theta)^{(n-1)}}{(n-1)!} f^n(a+\theta h)$$ is known as

a. Lagrange's remainder b. Cauchy's remainder
c. Maclaurin's remainder d. Taylor's remainder

(ii) Which of the following pair of functions do not satisfy the Cauchy's Mean Value Theorem in the interval in [-2,2] ?

a. $x^2,\ logx$ b. $sinx^2,\ x$
c. $|x-4|,\ x^2+4$ d. $x^2+1,\ \frac{x}{x^2+4}$

(iii) If $\sum_{n=1}^{\infty} x_n$ is convergent, then the series $\sum_{n=1}^{k} u_n + \sum_{n=1}^{\infty} x_n$ is

a. convergent b. divergent
c. oscillatory d. nothing can be said

(iv) For an odd function, the Fourier series expansion contains

a. only cosine terms b. only sine terms
c. both sine and cosine terms d. none of these

(v) $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x^2-y^2}{x^2+y^2} =$

a. 0 b. 1

c. $\frac{1}{2}$ d. none of these

(vi) The value of $\int_C (xdx - dy)$, where C is the line joining (0,1) to (1,0) is

a. $\frac{3}{2}$ b. $\frac{1}{2}$

c. 0 d. $\frac{2}{3}$

(vii) The value of the integral $\int_C xdy$ where C is the arc cut off from the parabola $y^2 = x$ from the point (0,1) to (1,-1) is

e. $-\frac{1}{3}$ f. $\frac{1}{3}$

g. 0 h. none of these

(viii) The value of $\int_0^{\frac{\pi}{2}} \int_0^{cos\theta} r^3 dr d\theta$ is

a. $\frac{\pi}{64}$ b. $\frac{3\pi}{64}$

c. $\frac{5\pi}{64}$ d. none of these

(ix) If $\vec{a}.(\vec{b} \times \vec{c}) = 0$, then the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are

a. independent b. coplanar

c. collinear d. none of these

(x) If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then $\nabla \times \vec{r} =$

a. 3 b. $\hat{\imath} + \hat{\jmath} + \hat{k}$

c. 0 d. none of these

Group – B

(Short Answer Type Questions) 3 x 5 = 15

Answer any *three* from the following :

2. Using Mean Value Theorem, prove that
$\dfrac{x}{1+x^2} < \tan^{-1} x < x,\ 0 < x < \dfrac{\pi}{2}$. 5

3. Show that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ is conditionally convergent. 5

4. If $z = \tan(y + ax) - (y - ax)^{\frac{3}{2}}$, prove that $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$. 5

5. Change the order of integration and hence evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dydx$. 5

6. Evaluate $\oint_C (e^x dx + 2ydy - dz)$ by Stokes' Theorem where C is the curve $x^2 + y^2 = 4,\ z = 2$. 5

Group – C

(Long Answer Type Questions) 3x 15 = 45

Answer any *three* from the following :

7. (a) Find the values of a, b and c if

$$\lim_{x\to 0} \frac{ae^x - bcosx + ce^{-x}}{xsinx} = 2 .$$ 5

(b) State Rolle's Theorem and explain it's geometrical significance. 5

(c) Prove that $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{cos\theta}} \times \int_0^{\frac{\pi}{2}} \sqrt{cos\theta} d\theta = \pi$. 5

8. (a) If $u_n = \frac{3n}{n+1}$, show that the sequence $\{u_n\}$ is monotonically increasing and bounded above. Is the sequence convergent? If yes, find the limit. 7

(b) Find the Fourier series expansion for the function $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$. Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$. 8

9. (a) If $u = \cos^{-1}\left(\frac{x^5 - 2y^5 + 6z^5}{\sqrt{ax^3 + by^3 + cz^3}} \right)$, then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -\frac{7}{2}\cot u.$$ 5

(b) Examine the following function for extreme values and saddle points :

$$f(x, y) = x^3 + 3xy^2 - 3y^2 - 3x^2 + 4$$ 10

10. (a) Find the area of the region enclosed by the hyperbola $xy = 16$, the lines $y = x,\ y = 0$ and $x = 8$. 8

(b) Show that $\iiint z^2 dxdy\, dz$ extended over the hemisphere $z \geq 0,\ x^2 + y^2 + z^2 \leq a^2$ is $\frac{2}{15}\pi a^5$. 7

11. (a) Show that $\overrightarrow{f} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational. Find a scalar function φ such that $\overrightarrow{f} = \vec{\nabla}\varphi$. 7

(b) Verify Greens's Theorem in the plane for $\oint_C [(y - sinx)dx + cosxdy]$ where C is the triangle whose vertices are (0, 0), $\left(\frac{\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, 1\right)$. 8
