BRAINWARE UNIVERSITY

Term End Examination 2018 - 19

Programme– B.Sc.(CS)

Course Name - Linear Algebra and Single Variable Calculus

Course Code - BMAT010602

(Semester – 1)

Time allotted:3 Hours **Full Marks : 70**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group –A

(Multiple Choice Type Questions) 10 x 1 = 10

1. ***Choose the correct alternative from the following:***

(i) The value of the determinant $\begin{vmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ 1 & 4 & 2 \end{vmatrix}$ is

a. -15 b. 15

c. 10 d. 13

(ii) The value of $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$ is

a. $\frac{2\pi}{\sqrt{3}}$ b. $\frac{3\pi}{\sqrt{2}}$

c. $\frac{\pi}{\sqrt{3}}$ d. $\frac{\pi}{\sqrt{2}}$

(iii) The value of $\beta\left(\frac{1}{2},\frac{1}{2}\right)$ is

a. π b. $\sqrt{\pi}$

c. $\frac{\sqrt{\pi}}{2}$ d. $\frac{\pi}{2}$

(iv) The sum of the eigen values of the matrix $A=\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ is

a. 6 b. 5

c. 4 d. 7

(v) If A is a non-null square matrix then $A+A^T$ is a

e. symmetric matrix f. skew-symmetric matrix

g. null matrix h. none of these

(vi) If $g(x)=\frac{1-x}{1+x}$ then $\mathrm{g}(\mathrm{x})+\mathrm{g}\left(\frac{1}{\mathrm{x}}\right)$ is

a. -1 b. 1

c. 2 d. 0

(vii) The value of $\Gamma(6)$ is

a. 720 b. 5

c. 6 d. 120

(viii) In the Mean-value theorem $f(h)=f(0)+hf'(\theta);\ \ 0<\theta<1,$ if $f(x)=\frac{1}{1+x}$ and $h=3$, then the value of θ is

a. 1 b. $\frac{1}{3}$

c. $\frac{1}{\sqrt{2}}$ d. $-\frac{1}{\sqrt{2}}$

(ix) The series $\sum_{n=1}^{\infty}\frac{n!.2^n}{n^n}$ is

a. convergent b. divergent

c. oscillatory d. none of these

(x) The modulus and amplitude of the complex number $4+4i$ are

a. $4\sqrt{2}$ and $\frac{\pi}{4}$ b. $4\sqrt{2}$ and $\frac{\pi}{2}$

c. $2\sqrt{2}$ and $\frac{\pi}{4}$ d. $2\sqrt{2}$ and $\frac{\pi}{2}$

Group – B

(Short Answer Type Questions) 3 x5 = 15

Answer any *three* from the following :

2. Evaluate $\lim_{x\to\infty}\dfrac{e^x - e^{-x} - 2x}{x - \sin x}$. 5

3. Using Lagrange's MVT, prove $0 < \dfrac{1}{x}\log\dfrac{e^x - 1}{x} < 1$. 5

4. Express $A = \begin{pmatrix} 1 & 3 & 4 \\ 7 & 2 & 6 \\ 2 & 8 & 1 \end{pmatrix}$ as P+Q, where P is a symmetric matrix and Q is a skew symmetric matrix. 3+2

5. Expand by Laplace method to prove that

$$\begin{vmatrix} a & b & c & d \\ -a & b & c & d \\ -a & -b & c & d \\ -a & -b & -c & d \end{vmatrix} = 8abc$$

5

6. Show that $\int_{-1}^{1}\dfrac{1}{x^3}dx$ exists in Cauchy's principal value sense but not in the general sense. 5

Group – C

(Long Answer Type Questions) 3x 15 = 45

Answer any *three* from the following :

7. (a) Determine the conditions under which the system of equations

$$x + y + z = 1$$
$$x + 2y - z = k$$
$$5x + 7y + az = k^2 \quad .$$

admits (i) only one solution (ii) no solution (iii) many solutions 7

(b) Using Laplace's method of expansion, prove that :

$$\begin{vmatrix} x & y & -u & -v \\ y & x & v & u \\ u & v & x & y \\ -v & -u & y & x \end{vmatrix} = (x^2 + v^2 - y^2 - u^2)^2$$

5

(c) If $x^2 + y^2 = 14xy$, prove that $2\log\left(\frac{x+y}{4}\right) = \log x - \log y$ 3

8. (a) If $A = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{pmatrix}$,show that $A^2 - 6A - 9I_3 = O$. Hence obtain a matrix B such that $BA = I_3$. 4

(b) Prove that the set S= $\left\{ \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix} : x_1 + x_2 = 0, x_i \in R \right\}$ is a subspace of the space of real matrices of size 2×2. Find a basis of S and hence determine the dimension of S. 5

(c) Is Rolle's Theorem applicable for the function $f(x) = (x-p)^m (x-q)^n, x \in [p,q]$, where m, n are positive integers? If so, find the constant c of Rolle's Theorem, where c has its usual meaning. 6

9. (a) Obtain the Fourier series to represent $f(x) = x^2$ in $-\pi \le x \le \pi$. Hence, show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots\ldots\infty = \frac{\pi^2}{8}$ 7

(b) Prove that $\beta(\mathrm{m,n}) = \int_0^\infty \frac{x^{m-1}}{(1+\mathrm{x})^{m+n}} dx$. 4

(c) If $z = x + iy$ and $|z+6| = |2z+3|$ prove that $x^2 + y^2 = 9$. 4

10. (a) Find Fourier sine series for $f(x) = e^x$ in $0 < x < \pi$. 4

(b) State Cayley-Hamilton Theorem and verify the same for the matrix A= $\begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}$. Hence find A^{-1} and A^8 . 6

(c) State D'Alembert's Ratio test. Using this test, examine the convergence of the following series $1 + \frac{2^a}{2!} + \frac{3^a}{3!} + \frac{4^a}{4!} \ldots\ldots\infty \ (a > 0)$ 2+3

11. (a) If $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, find the rank of the matrix $A + A^2$ 5

(b) Evaluate $\lim_{x \to \infty} \left\{ \frac{1}{n+1} + \frac{1}{n+2} + \cdots\cdots + \frac{1}{2n} \right\}$ 5

(c) State Maclaurin's theorem for infinite series. Expand the functions e^{ax} in powers of x in an infinite series. 5
