BRAINWARE UNIVERSITY

Term End Examination 2019 – 20

Programme – Master of Science in Mathematics

Course Name – Real Analysis

Course Code: MSCMC102

(Semester – 1)

Time allotted: 2 Hours 30 Minutes **Full Marks: 60**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group –A

(Multiple Choice Type Question) 20 x 1 = 20

1. Answer any *twenty* from the following

(i) Let X be a connected subset of real numbers. If every element of X is irrational, then the cardinality of X is

a. infinite
b. countably infinite
c. 2
d. 1

(ii) Consider the following sets of function on R

W= The set of constant functions on RR

X= The set of polynomial functions on RR

Y= The set of continuous functions on RR

Z= The set of all functions on RR

Which of these sets has the same cardinality as that of R?

a. Only W
b. Only W and X
c. Only W,X and Z
d. Only W,X,Y and Z

(iii) Which of the following sets of the functions are uncountable?

a. $\{f|f:N\to \{1,2\}\}$
b. $\{f|f: \{1,2\}\to N\}$
c. $\{f|f: \{1,2\}\to N, f(1)\leq f(2)\}$
d. $\{f|f:N\to \{1,2\}, f(1)\leq f(2)\}$

(iv) Let X be the set roots of unity in CC. Let S(X) be the set of all sequences of elements in X. Which of the following subset of S(X) is countable?

a. The set A of all (xn)∈S(X) such that (xn) is an eventually constant sequence .

b. The set B of all (xn)∈S(X) such that xn=1, whenever n is prime number.

c. The set C of all (xn)∈S(X)such that each xn is 26th root of unity .

d. The set D of all (xn)∈S(X) such that x2n=1 for all n≥≥ 1 .

(v) If $\sum_{n=1}^{\infty} x_n$ is convergent, then the series $\sum_{n=1}^{k} u_n + \sum_{n=1}^{\infty} x_n$ is

a. convergent

b. divergent

c. oscillatory

d. nothing can be said

(vi) The series $\sum_{n=1}^{\infty} \frac{1}{n^{(p+1)}}$ is divergent if

a. $p \leq 0$

b. $p > 1$

c. $p > 0$

d. $p \leq 1$

(vii) Let $f(x) = x - \frac{x^3}{3^2.2!} + \frac{x^5}{5^2.4!} - \frac{x^7}{7^2.6!}$ Find a point nearest to such that c

a. f'(c) = 1

b. 1

c. 0

d. $2.3445 * 10^{-9}$

(viii) Which of the following is not true for a set in R?

a. A set may not have an infimum in R

b. Infimum of a set may not belong to the set.

c. Infimum and supremum of a set may be equal.

d. Supremum of a bounded below set always exists in R.

(ix) Which algebraic property is not true for the set of real numbers R?

a. For all a ≠ 0, b ∈ R such that a.b = 1 implies b = 1/ a.

b. a. (1/ a) = 1 for all a ≠ 0.

c. √a2 = a for all a ∈ R

d. If a.b > 0 then either a > 0 and b > 0 or a < 0 and b < 0

(x) Which of the following sequences is convergent?

a. (n)

b. $((-1)^n)$

c. $(\frac{\sin n}{n})$

d. None of these.

(xi) Let $a_n = \frac{4^{3n}}{3^{4n}}$ .Then the sequence (an)

a. is unbounded.

b. is bounded but not convergent

c. converges to 0.

d. converges to 1.

(xii) A number which is neither even nor odd is

a. 0 b. 2

c. 2n such that $n \in Z$ d. 2π

(xiii) If a real number is not rational then it is ……………

a. integer b. algebraic number

c. irrational number d. complex numbers

(xiv) Let $A=\{x|x\in N \wedge x2\leq 7\}$. Then supremum of A is

a. 7 b. 3

c. 0 d. Does not exist

(xv) A sequence $\{(-1)^n\}$ is

a. convergent. b. unbounded.

c. divergent. d. bounded.

(xvi) If the sequence is decreasing, then it …………….

a. converges to its infimum. b. diverges.

c. may converges to its infimum d. is bounded.

(xvii) If the sequence is increasing, then it …………….

a. converges to its supremum. b. diverges.

c. may converges to its supremum. d. is bounded.

(xviii) If a sequence {an} is convergent then the series ∑an …………….

a. is convergent. b. is divergent.

c. may or may not convergent d. None of these

(xix) An series ∑an is said to be absolutely convergent if

a. |∑an| is convergent. b. |∑an| is convergent but ∑an is divergent.

c. ∑|an| is convergent. d. ∑|an| is divergent but ∑an∑an is convergent.

(xx) A number L is called limit of the function f when x approaches to c if for all $\varepsilon>0$, there exist $\delta>0$ such that ……… whenever $0<|x-c|<\delta$.

a. $|f(x)-L|>\varepsilon$ b. $|f(x)-L|<\varepsilon$

c. $|f(x)-L|\leq\varepsilon$ d. $|f(x)-L|\geq\varepsilon$

(xxi) If $\lim_{x}\to cf(x)=L$, then ……….. sequence $\{ x_n \}$ such that $x_n \to c$, when $n\to\infty$, one has $\lim_{n}\to\infty\ f(x_n)=L$.

a. for some b. for every

c. for few d. None of these

(xxii) Which one is not partition of interval [1,5].

a. {1,2,3,5}
b. {1,3,3.5,5}
c. {1,1.1,5}
d. {1,2,3,4,5}

(xxiii) Let $f:[2,4]\longrightarrow R$ be a continuous function such that f(2)=3 and f(4)=6. The most we can say about the set f([2,4]) is that

a. It is a set which contains [3,6].
b. It is a closed interval.
c. It is a set which contains 3 and 6.
d. It is a closed interval which contains [3,6].

(xxiv) Let A be a set. What does it mean for A to be countable?

a. One can assign a different element of A to each natural number in N
b. There is a way to assign a natural number to every element of A such that each natural number is assigned to exactly one element of A
c. A is of the form $\{a_1,a_2,a_3,\dots\}$ for some sequence $a_1,a_2,a_3,\dots$
d. One can assign a different natural number to each element of A

(xxv) A and B be bounded non-empty sets. Following are two groups of statements:

a. $\inf(A)\leq \inf(B)$
b. $\inf(A)\leq \sup(B)$
c. $\sup(A)\leq \inf(B)$
d. $\sup(A)\leq \sup(B)$

Group – B

(Short Answer Type Questions) 4 x 5 = 20

2. Examine the following function for continuity at origin: 5

$$f(x) = \begin{cases} \dfrac{xe^{\frac{1}{x}}}{1+e^{\frac{1}{x}}} & if\ x \neq 0 \\ 0\ if\ x = 0 \end{cases}$$

3. Define a Cauchy Sequence and also state the Cauchy Convergence Criterion 5

4. (a) Test the convergence of the series $\sum \frac{(n^3+1)^{\frac{1}{3}}-n}{logn}$ 3

(b) Show that the function f(x) = $x^2$is differentiable on [0,1] 2

5. Use Squeeze Theorem to determine the limit of the following sequences: 5

(a) $\{u_n\}n^{\frac{1}{n^2}}$

(b) $n!^{\frac{1}{n^2}}$

6. Define Supremum and Infimum of a set. Also give examples. 5

7. Let (X, d) be a metric space. 5

   (a) Define the open ball $B_r(x)$ of radius $r > 0$ and center $x \in X$.

   (b) Define an open set $A \subset X$.

   (c) Show that the open ball $B_r(x) \subset X$ is an open set.

Group – C

(Long Answer Type Questions) 2 x 10 = 20

Answer any *two* from the following

8. Every compact subset F of a metric space (X,d) is closed. 10

9. Prove that a function f is integrable with respect to α on [a,b] if and only if for every $\varepsilon > 0$, there exists a partition P of [a,b] such that $U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$. 10

10. Prove that a necessary and sufficient condition for the convergence of a sequence $\{S_n\}$ is the, for each $\varepsilon > 0$ there exists a positive integer m such that $|S_{n+p} - S_n| < \varepsilon, \forall n \geq m \wedge p \geq l$ 10

11. State and prove Taylor's theorem. 10
