BRAINWARE UNIVERSITY

Term End Examination 2019 – 20

Programme – Master of Science in Mathematics

Course Name – Ordinary Differential Equations

Course Code: MSCMC104

(Semester – 1)

Time allotted: 2 Hours 30 Minutes **Full Marks: 60**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group –A

(Multiple Choice Type Question) 20 x 1 = 20

1. Answer any *twenty* from the following

(i) Determine the order of the differential equation $\frac{d^3y}{dx^3} - 5x\frac{dy}{dx} = e^x + 1$

a. Third order
b. Second order
c. First order
d. None of these

(ii) Determine the unknown function and independent variable of $t\ddot{y} + t^2y - (sint)\sqrt{y} = t^2 - t + 1$

a. y, t
b. t, y
c. t, sint
d. None of these.

(iii) The value of Lipschitz constant of $f(x,y) = x^3 siny, D: |x| \leq 2, -\infty < y < \infty$ is

a. 8
b. 24
c. 9
d. 40

(iv) $y = \ln x$ is a solution of $xy'' + y = 0$ in

a. $(0, \infty)$
b. $(-\infty, \infty)$

(v) The solution of the initial value problem $y'' + 4y = 0$ given $y(0) = 0$ and $y'(0) = 1$

a. $y = sin2x$
b. $y = x$
c. $y = \frac{1}{2}sin2x$
d. None of these.

(vi) Which of the following functions are solutions of the differential equation $y' - 3y = 6$?

a. $y = -2$
b. $y = e^{3x} - 2$
c. $4e^{3x} - 2$
d. All a,b,c

(vii) The value of C_1 and C_2 so that $y(x) = C_1e^{2x} + C_2e^{2x} + 2sinx$ will satisfy the conditions $y(0) = 0$ and $y'(0) = 1$

a. $C_1 = -1, C_2 = 1$
b. $C_1 = 1, C_2 = 1$
c. $C_1 = -1, C_2 = -1$
d. $C_1 = -1, C_2 = 2$

(viii) Which of the following functions are solutions of the differential equation $y'' - xy' + y = 0$?

a. y = x, y = 0
b. y = x
c. y = 0
d. none of these

(ix) Picard's iteration formula is

a. $y^{(n+1)}(x) = y_0 + \int_{x_0}^{x} f(x, y^{(n)}(x))dx$
b. $y^{(n+1)}(x) = y_0 + \int_{x_0}^{x} f(x, y^{(n-1)}(x))dx$
c. $y^{(n+1)}(x) = y_0 + \int_{x_0}^{x} f(x, y^{(n+1)}(x))dx$
d. None of these.

(x) Which of the following differential equation is non-linear?

a. $yy''' + xy' + y = x^2$
b. $y'' - y = 0$
c. $3y' + xy = e^{-x^2}$
d. None of these.

(xi) Find the wronskian of the set $\{e^x, e^{-x}\}$

a. -2
b. -3
c. -4
d. -6

(xii) The set $\{x, x^2, 1\}$ is ………… on $(-\infty, \infty)$

a. Linearly dependent
b. Linearly independent

(xiii) The general solution of $y'' - 2y' + y = 0$ if it is known that two solutions are $y_1(x) = e^{-x}$ and $y_2(x) = 5e^{-x}$

a. $y(x) = c_1e^{-x} + c_2e^{-x}$
b. $y(x) = c_1e^{-x} + c_25e^{-x}$
c. impossible
d. None of these.

(xiv) All solutions of the initial value problem $y'' + e^{-x}y' + (x + 1)y = 0$ given $y(1) = 0, y'(1) = 0$

a. y = 0
b. $c_1e^{-x} + c_25e^{-x}$
c. $c_1e^{-x} + c_2e^{-x}$
d. None of these.

(xv) In the differential equation $y'' + 3y' + 2xy = 0$, the point x = 1 is

a. Ordinary point
b. Singular point
c. Regular Singular point
d. None of these.

(xvi) In the differential equation $(x + 1)y'' + \frac{1}{x}y' + xy = 0$, the point x = 0 is

a. Ordinary point
b. Singular point
c. Regular Singular point
d. None of these.

(xvii) $P_n(-1) =$

a. 1
b. 0
c. $(-1)^n$
d. None of these

(xviii) A solution to a boundary value problem which satisfies boundary condition is a solution of the

a. Integral equation
b. Differential equation
c. Maxwell's equation
d. logical equation

(xix) Wronskian in a set of solution represent its

a. superposition
b. linear independency
c. combinations
d. integrations

(xx) The value of $J_{\frac{1}{2}}(x)$ is

a. $\sqrt{\frac{2}{\pi x}}\ cosx$
b. $\sqrt{\frac{2}{\pi x}}\ sinx$
c. $\sqrt{\frac{2x}{\pi}}\ cosx$
d. None of these.

(xxi) The form of Rodrigues' formula is

a. $P_n(x) = \frac{1}{2^n.n!}\frac{d^n}{dx^n}(x^2 - 1)^n$
b. $P_n(x) = \frac{1}{2^n.n!}\frac{d^n}{dx^n}(x^2 - 1)^{n+1}$
c. $P_n(x) = \frac{1}{2^n.n!}\frac{d^n}{dx^n}(x^3 - 1)^n$
d. None of these

(xxii) The eigen values of Sturm-Liouville problem are all

a. real and non-negative
b. real and negative
c. cannot be decided
d. None of these

(xxiii) A nonhomogeneous problem has a unique solution when and only when the associated homogeneous problem has

a. unique solution
b. infinite solution
c. no solution
d. None of these.

(xxiv) Which of the following equation is homogeneous?

a. $y' = \frac{y+x}{x}$

b. $y' = \frac{y^2}{x}$

c. $y' = \frac{y+x^2}{x^3}$

d. None of these

(xxv) A separable differential equation is always exact.

a. true

b. false

Group – B

(Short Answer Type Questions) 4 x 5 = 20

2. Let *f* be a real function defined on a domain D of the *xy*-plane. State a sufficient condition for the function *f* to satisfy a Lipschitz condition in D. Is the condition also necessary? Justify your answer with an example. 5

3. Discuss the existence and uniqueness of solution of the initial-value problem 5

$(x^2-4)\frac{d^4y}{dx^4} + 2x\frac{d^2y}{dx^2} + (sinx)y = 0,$

$y(0) = 0,\ y'(0) = 1,\ y''(0) = 1,\ y'''(0) = -1.$

Find the interval where it is defined.

4. (a) Define a bounded function with suitable example. 2+1

(b) Write the difference between a domain and a closed domain. 2

5. (a) State the Sturm-Liouville problem. 3

(b) Find the interval in which the series solution to the Legendre's equation will converge. 2

6. Prove that $\int_{-1}^{1} P_m(x)\, P_n(x) dx = 0\ (m \neq n)$ 5

7. Find the indicial equation of $x^2y'' + (x^2+2x)y' - 2y = 0$ near x=0 5

Group – C

(Long Answer Type Questions) 2 x 10 = 20

Answer any *two* from the following

8. State the Existence and Uniqueness theorem for the solution of the initial value problem 10

$\frac{dy}{dx} = f(x,y), y(x_0) = y_0.$

Discuss the method of successive approximations and give an outline of the main steps for the proof of the above theorem.

9. State and prove the Existence and Uniqueness Theorem for the initial value problem associated with an nth order differential equation of the form 10

$$\frac{d^n y}{dx^n} = f\left(x, y, \frac{dy}{dx}, \dots, \frac{d^{n-1}y}{dx^{n-1}}\right).$$

10. Consider the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

(a) Show that e^x and xe^x are linearly independent solutions of this equation on the interval $-\infty < x < \infty$. 4

(b) Write the general solution of the given equation. 1

(c) Find the solution that satisfies the conditions $y(0) = 2, y'(0) = 4$. Explain why the solution is unique. Over which interval is it defined? 5

11. Show that the theorem stated in part (a) guarantees the existence of a unique solution of the initial value problem $\frac{dy}{dx} = x^2 + y^2, y(0) = 0$ on the interval $|x| \leq \sqrt{2}/2$. 10
