BRAINWARE UNIVERSITY

Term End Examination 2024-2025
Programme – B.Tech.(ME)-2024/B.Tech.(EE)-2024
Course Name – Engineering Mathematics -I
Course Code - BBS00011
(Semester I)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Select the correct value of $\beta\left(\frac{1}{2},\frac{1}{2}\right)$

a) π b) $\sqrt{\pi}$

c) $\frac{\sqrt{\pi}}{2}$ d) $\frac{\pi}{2}$

(ii) Identify the value of $\int_0^\infty e^{-x^2}dx$

a) π b) $\sqrt{\pi}$

c) $\frac{\sqrt{\pi}}{2}$ d) $\frac{\pi}{2}$

(iii) For $k>0, n>0$, identify the value of $\int_1^\infty \frac{(\log y)^{n-1}}{y^{k+1}}dy$

a) $\frac{\Gamma(n)}{k^n}$ b) $\frac{\Gamma(k)}{k^n}$

c) $\frac{\Gamma(k)}{n^n}$ d) None of these

(iv) Select the correct value of $\beta(1,1)$

a) π b) 1

c) $\frac{\sqrt{\pi}}{2}$ d) None of these

(v) Select the value of $\lim_{x\to 0^+} x^{-x}$

a) 1
b) 0
c) 2
d) None of these

(vi) Choose the correct option. The integral $\int_1^\infty \frac{1}{x(\ln x)^2}dx$ is:

a) Finite but not evaluated
b) Undefined
c) Convergent
d) Divergent

(vii) Choose the correct option

a) The rank of a matrix is number of rows in the matrix
b) The rank of a matrix is number of columns in the matrix
c) The rank of a matrix is maximum value in the matrix
d) The rank of a matrix is maximum number of linearly independent rows or columns in the matrix

(viii) Choose the eigenvalues of a 2x2 rotation matrix:

a) Complex numbers in the form cos(θ) ± i sin(θ)
b) 1 and 1
c) -1 and -1
d) 0 and 1

(ix) Select the correct option in a vector space, the zero vector is unique because:

a) It can be represented as a linear combination of other vectors
b) It has a magnitude of zero.
c) It is the additive identity and has no other representation.
d) It is orthogonal to all other vectors in the space.

(x) The series $\sum_{n=1}^{\infty} \frac{1}{n(p+1)}$ is divergent then select the correct value of p

a) $p \leq 0$
b) $p > 1$
c) $p > 0$
d) $p \leq 1$

(xi) Examine the type of the series 1+2+3+...

a) Convergent
b) Divergent
c) Absolutely convergent
d) None of these

(xii) Calculate $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2} =$

a) 0
b) 1
c) $\frac{1}{2}$
d) None of these

(xiii) Select the following that is not a property of the Fourier Transform

a) Linearity
b) Time-shifting
c) Differentiation
d) Modulation

(xiv) Select the correct one: The Fourier transform of the Dirac delta function is

a) 0
b) 1
c) Infinity
d) None of these

(xv) Identify the correct one: The Laplace Transform of 1 is

a) $\frac{1}{s}$
b) $\frac{1}{s^2}$
c) s
d) 1

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Explain the Time Scaling property of the Fourier Transform with an example. (3)

3. Define the Maclaurin Series expansion of sin x. (3)

4. Establish that $\lim_{n\to\infty} \sqrt{n+1} - \sqrt{n} = 0$ (3)

5. Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{2}{3^n - 1}$ (3)

6. If $z = x^2 - y^2$, $x = \sin(t)$, $y = \cos(t)$, Evaluate $\frac{dz}{dt}$ where $t = \pi/3$. (3)

OR

If $f(x,y) = \frac{x+y}{1-xy}$ and $g(x,y) = \tan^{-1} x + \tan^{-1} y$ Evaluate $\frac{\partial(f,g)}{\partial(x,y)}$. (3)

Group-C
(Long Answer Type Questions) 5 x 6=30

7. Recognize that $\frac{(b-a)}{\sqrt{1-a^2}} < sin^{-1} b - sin^{-1} a < \frac{(b-a)}{\sqrt{1-b^2}}$ if $0 < a < b < 1$ (5)

8. Illustrate $\int_0^{\infty} e^{-x^4} x^2 dx * \int_0^{\infty} e^{-x^4} dx = \frac{\pi}{8\sqrt{2}}$ (5)

9. Discover the interval and radius of convergence $\sum \frac{(x-2)^n}{(n+1)n^3}$ (5)

10. Established that the function $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq 0 \\ 0, (x,y) = 0 \end{cases}$ repeated limits are equal but simultaneous limit does not exist. (5)

11. Evaluate the inverse of the matrix $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}$ (5)

12. Define T: $R^2 \to R^2$ by T ((x, y)) = (x + y, x − y). Then evaluate T^{-1}. (5)

OR

Evaluate the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$. (5)

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