BRAINWARE UNIVERSITY

Term End Examination 2024-2025
Programme – M.Sc.(MATH)-2024
Course Name – Linear Algebra
Course Code - MSCMC101
(Semester I)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) If $T : V \to W$ be a linear mapping, then identify the correct option

a) dim (Ker T) + dim (Im T) = dim (V)
b) dim (Ker T) + dim (Im T) = dim (W)
c) dim (Ker T) + dim (Im T) = 3
d) None of these

(ii) Let $T : R^3 \to R^2$ be a linear transformation defined by $T(x,y,z) = (x-y, x-z)$, then identify the dimension of the nullspace of T

a) 0
b) 1
c) 2
d) 3

(iii) If S is a subspace of a vector space $(V,+,.)$ over R, where R is the set of all real numbers. Then select the true statement.

a) $\alpha+\beta \in S$ *whenever* $\alpha, \beta \in S$
b) $\alpha+2\beta \in S$ *whenever* $\alpha, \beta \in S$
c) $-\alpha+\beta \in S$ *whenever* $\alpha, \beta \in S$
d) All of a, b, c is true.

(iv) Let V be a vector space over the set of all real numbers R. Let θ be the zero vector of V. Then identify $2.\theta = ?$

a) 1
b) θ
c) 2
d) None of these

(v) Let A and B be two subspaces of a vector space V, then select the correct statement

a) $A \cap B$ is a subspace of V.
b) both $A \cap B$ and $A \cup B$ are subspaces of V.
c) $A \cup B$ is a subspace of V.
d) neither $A \cap B$ nor $A \cup B$ are subspaces of V.

(vi) Select the correct option: A vector space V is finite dimensional if it has

a) finite basis
b) finite elements
c) no basis
d) None of the above

(vii) Let A and B be square matrices of order 3. If $|A|=3$ and $|B|=-1$ then examine $|2A\cdot 4B|=?$

a) $(-4)2^3$
b) $(-3)2^3$
c) $(-3)2^9$
d) $(-4)2^9$

(viii) If $\alpha=(1,2,3,4)$ and $\beta=(2,0,-3,1)$ then examine $\|\alpha-\beta\|=$

a) $\sqrt{30}$
b) $3\sqrt{30}$
c) $\sqrt{38}$
d) None of these

(ix) If $\lambda\neq 0$ is an Eigen value of a matrix A then compute $\det(A-\lambda I)=$

a) λ
b) $-\lambda$
c) 2λ
d) 0

(x) Choose the correct option: Any set of linearly independent vectors can be orthonormalized by the

a) Cramer's rule
b) Sobolev Method
c) Gram-Schmidt procedure
d) Pound-Smith procedure

(xi) If $\lambda=1$ is an Eigen value of the matrix $\begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$ then evaluate the corresponding Eigen vector

a) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$
b) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$
c) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
d) $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$

(xii) If $V=R^3$ be equipped with inner product $(x,y)=x_1y_1+x_2y_2+x_3y_3$. Then select the set of vectors that are linearly independent.

a) $\{(0,1,0),(0,0,1),(-1,0,1)\}$
b) $\{(0,1,0),(0,-1,0),(0,0,1)\}$
c) $\{(0,1,0),(0,0,1),(-1,0,1)\}$
d) $\{(1,0,1),(0,1,0),(-1,0,1)\}$

(xiii) Choose the right option: The quadratic form $Q(x, y, z) = (x - y)^2 + 3z^2$ is

a) positive definite

b) positive semidefinite

c) Indefinite

d) None of these

(xiv) The leading principal minors of a 4 by 4 matrix are $|A1| = -1$, $|A2| = 1$, $|A3| = 2$, and $|A4| = |A| = 0$. Then select the correct statement

a) the matrix is negative semidefinite

b) its definiteness cannot be determined with this information

c) the matrix is indefinite

d) None of these

(xv) Select the correct option: The leading principal minors of a 4 by 4 matrix are $|A1| = -1$, $|A2| = 1$, $|A3| = -2$, and $|A4| = |A| = 1$. Then, the matrix is

a) negative definite

b) indefinite

c) positive definite and negative definite

d) None of these

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Illustrate that $q(Y) = ax^2 + bxy + cy^2$ is positive definite if and only if $a>0$ and the discriminant $D = b^2 - 4ac < 0$. (3)

3. Identify the characteristic polynomial of $\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$. (3)

4. Identify the row rank of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$. (3)

5. Explain the statement, "The matrix $A = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ is not diagonalizable." (3)

6. Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Formulate a matrix P such that $P^{-1}AP$ is diagonal. (3)

OR

Justify the statement, "The quadratic form $x^2 + 2y^2 + 3z^2 - 2xy + 4yz$ is indefinite." (3)

Group-C
(Long Answer Type Questions) 5 x 6=30

7. Use Gaussian elimination to construct the solution for the given system of equations: $3x+y-z=1,\ x-y+z=-3,\ 2x+y+z=0$. (5)

8. Let V be a vector space over the field F. Then justify that intersection of any collection of subspace of V is also a subspace of V. (5)

9. Examine whether the set of vectors formed by the matrices A, B and C are dependent where $A=\begin{bmatrix}1 & 2\\ 3 & 1\end{bmatrix}, B=\begin{bmatrix}3 & -1\\ 2 & 2\end{bmatrix}$ and $C=\begin{bmatrix}1 & -5\\ -4 & 0\end{bmatrix}$ (5)

10. Describe the quadratic form to normal form of $xy+yz+zx$. Then evaluate the rank and signature of the same. (5)

11. If α and β be any two vectors in an inner product space $(R^2, \|.,.\|)$, then illustrate that $\|\alpha+\beta\| \le \|\alpha\|+\|\beta\|$. (5)

12. Justify that set of all symmetric matrices of order $n \times n$ form a subspace of the vector space of all $n \times n$ matrix over the field F. (5)

OR

Explain elementary row transformation on a matrix and with the help of these, solve $AX=0$ where $A=\begin{bmatrix}3 & -1 & 2\\ 2 & 1 & 1\\ 1 & -3 & 0\end{bmatrix}$. (5)

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