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BRAINWARE UNIVERSITY

Term End Examination 2024-2025
Programme – M.Sc.(MATH)-2024
Course Name – Real Analysis
Course Code - MSCMC102
(Semester I)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Select the correct option. If x is a real number, then there exists a natural number n such that

a) $n < x$
b) $n > x$
c) $n = |x|$
d) None of these

(ii) Identify the set of convergence of the series $\sum_{k=1}^{\infty} k^k x^k$.

a) $[0,1]$
b) $[0,1)$
c) $\{0,1\}$
d) $\{0\}$

(iii) Select the correct option. Let $\sum_{k=0}^{\infty} a_k (x-c)^k$ be a power series with radius of convergence R. If $R = \infty$, then

a) the series converges only at c.
b) the series converges absolutely for all x.
c) the series converges absolutely for all x satisfying $|x - c| < R$.
d) None of these

(iv) Write the correct expansion for $\frac{x}{1+x^2}, -1 < x < 1$

a) $\sum_{k=1} (-1)^k x^{2k+1}$
b) $\sum_{k=0} (-1)^k x^{2k+1}$
c) $\sum_{k=1} (-1)^{k+1} x^{2k+1}$
d) None of these

(v) Write the correct option. $2 \log 2$ is represented by the series

a) $\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}-\ldots$

b) $-\frac{1}{1.2}+\frac{1}{2.3}-\frac{1}{3.4}+\ldots$

c) $1+\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}-\ldots$

d) $-1+\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}-\ldots$

(vi) Write the correct option. A function f is defined on $\left(-\frac{1}{3},\frac{1}{3}\right)$ by $f(x) = 1 + 2.3x + 3.3^2x^2 + \cdots + n.3^{n-1}x^{n-1} + \cdots$. Then f is

a) continuous on $\left(-\frac{1}{3},\frac{1}{3}\right)$

b) continuous on $\left(-\frac{1}{3},\frac{1}{3}\right]$

c) continuous on $\left[-\frac{1}{3},\frac{1}{3}\right)$

d) None of these

(vii) Select the correct option. If a vector space X is spanned by 10 vectors, then

a) dim X $\le$ 10

b) dim X $\ge$ 10

c) dim X $\ge$ 11

d) None of these

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(viii) Select the correct option. If $A \in L(R^n, R^m)$, then

a) $\infty \ge \|A\| > 0$

b) $\infty > \|A\| > 0$

c) $\infty > \|A\| \ge 0$

d) $\infty > \|A\| > -\infty$

(ix) Select the correct option. If a function F(x) is the integral of another function f(x) with respect to x, then:

a) $F'(x) = f(x)$

b) $F'(x) = 1/f(x)$

c) $F'(x) = \int f(x)\,dx$

d) $F'(x) = f'(x)$

(x) Select a curve from the following curves that is not rectifiable.

a) $y = x^{\frac{3}{2}}$, for x in [0, 1]

b) $y = \frac{1}{x}$, for x in [1, ∞)

c) $y = cos(x)$, for x in [0, 2π]

d) $y = e^{-x}$, for x in [0, ∞)

(xi) Select the correct option. To integrate a vector-valued function over an interval [a, b] means

a) Finding the magnitude of the vector at points in [a, b].

b) Finding the average of all vector values in [a, b].

c) Finding the area under the curve traced by the vector-valued function in [a, b].

d) Finding the tangent to the vector-valued function at points in [a, b].

(xii) Select the correct option. A curve $\gamma : [a, b] \to R^k$ is rectifiable if

a) γ' exists in $[a, b]$

b) γ' exists and continuous in $[a, b]$

c) $\gamma'(a) = \gamma'(b)$

d) None of these

(xiii) Select the length of the curve $\gamma(t) = (\cos t, \sin t), t \in [0, 2\pi]$.

a) π

b) 2π

c) 0

d) None of these

(xiv) If f is continuous on $[a, b]$, then select the integral that does not exist

a) $\int_a^b f(x)d(x^2)$

b) $\int_a^b f(x)d(2+x)$

c) $\int_a^b f(x)d(x^2+x)$

d) None of these

(xv) Let $f(x) = [x], x \in [0,3]$. Select the value of $\int_0^3 f(x)dx$

a) 1 b) 2

c) 3 d) 4

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Establish that $\frac{\pi^2}{9} < \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x}{\sin x} dx < \frac{2\pi^2}{9}$. (3)

3. Let $f(x)$ be the sum of the power series $\sum_{n=0}^{\infty} a_n x^n$ on $(-R, R)$ for some $R > 0$. If $f(x) = f(-x)$ for all $x \in (-R, R)$, deduce that $a_n = 0$ for all odd n. (3)

4. Let a function $f: [a, b] \to \mathbb{R}$ be integrable on [a, b]. Then $|f|$ is integrable on [a, b]. Examine whether the converse is true or not. (3)

5. Examine that $\lim_{n\to\infty} \left(\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}}\right) = 1$ (3)

6. Let A and B be two bounded subsets of **R** such that $x \in A, y \in B \Rightarrow x \leq y$. Justify that $supA \leq infB$. (3)

OR

(3)

Let $f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}, (x,y) \neq (0,0)$
$= 0, (x,y) = (0,0)$

Write the value of $f_{xy}(0,0)$ and $f_{yx}(0,0)$.

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Group-C

(Long Answer Type Questions) 5 x 6=30

7. Show that every sequence of real numbers has a monotone subsequence. (5)

8. Assuming the expansion $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ for $-1 < x \leq 1$, then deduce that $\int_0^1 \frac{\log(1+x)}{x} dx = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$. (5)

9. Let $f(x) = [x], x \in [0,3]$. Deduce that f is integrable on [0,3]. Evaluate $\int_0^3 f$. (5)

10. If S be a non-empty compact subset of $\mathbb{R}$, then examine that $\sup S$ and $\inf S$ belong to S. (5)

11. Let $c \in \mathbb{R}$ and a real function f be such that f'' is continuous on some neighbourhood of c. Justify that $\lim_{h \to 0} \frac{f(c+h)-2f(c)+f(c-h)}{h^2} = f''(c)$. (5)

12. Let a function $f: [a, b] \to \mathbb{R}$ be integrable on [a, b]. If M be the supremum and m be the infimum of f on $[a, b]$ then justify that (5)

$$m(b-a) \le \int_a^b f \le M(b-a)$$

OR

Let $f: [a, b] \to \mathbb{R}$ be continuous on [a, b]. Then justify that there exists a point $c \in [a, b]$ such that $\int_a^b f = f(c)(b-a)$. (5)

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