BRAINWARE UNIVERSITY

Term End Examination 2024-2025
Programme – M.Tech.(CSE)-AIML-2024
Course Name – Mathematics - I
Course Code - MBS00001
(Semester I)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Select the correct example of qualitative data from the following.

a) Height of students
b) Temperature of a substance
c) Color of cars
d) Number of books on a shelf

(ii) Examine the mean of the following data set: 4, 8, 6, 5, 7.

a) 5
b) 6.5
c) 6
d) 7

(iii) Classify the skewness of the data set if the mean is 20, median is 22, and mode is 24.

a) Positive Skewness
b) Negative Skewness
c) Zero Skewness
d) Normal Distribution

(iv) Number of times each value appears is called value's _____. Select the correct option.

a) Range
b) Frequency
c) Mode
d) Standard deviation

(v) In a Binomial (n, p) distribution, if its mean and variance are 2 and 16/9 respectively, then compute the values of n and p.

a) $18,\frac{1}{9}$
b) $16,\frac{1}{9}$
c) $16,\frac{1}{8}$
d) $18,\frac{1}{8}$

(vi) Identify one of the conditions for a counting process $\{N(t), t \geq 0\}$, is said to be Poisson process if-

a) The process has non-stationary
b) The process has stationary and independent increments
c) The process has dependent increments
d) The process is involuntary

(vii) A transition probability matrix P is said to be doubly stochastic if _____. Classify the correct option.

a) $\sum_j P_{ij} = 1, \forall i$

b) $\sum_i P_{ij} < 1, \forall j$

c) $\sum_i P_{ij} = 0, \forall j$

d) $\sum_i P_{ij} = 1, \forall j$

(viii) If X is normally distributed with zero mean and unit variance, then compute the expectation of X^2.

a) 1

b) 2

c) 8

d) 0

(ix) Let X(t) and Y(t) be two random processes with respective auto correlation functions $R_{xx}(\tau)$and $R_{yy}(\tau)$. Then compute $|R_{xy}(\tau)|$

a) $= \sqrt{R_{xx}(0)R_{yy}(0)}$

b) $\geq \sqrt{R_{xx}(0)R_{yy}(0)}$

c) $\leq \sqrt{R_{xx}(0)R_{yy}(0)}$

d) $> \sqrt{R_{xx}(0)R_{yy}(0)}$

(x) Write the definition of a Markov chain:

a) A stochastic process where the future state depends only on the present state.

b) A deterministic process with known future states.

c) A process where past states influence future states.

d) A sequence of random variables that are independent of each other.

(xi) Analyze the correct option for the claim: "A Markov chain with an absorbing state will always reach that state eventually."

a) True

b) False

c) True only for finite Markov chains

d) False only for continuous Markov chains

(xii) The steady-state probability vector π of a discrete Markov chain with transition probability matrix P satisfies the matrix equation of _____. Select the correct option.

a) $P\pi = 0$

b) $(2 - P)\pi = 0$

c) $P\pi = \pi$

d) $P^t\pi = 0$

(xiii) Select the correct formula for Bayes' theorem

a) $P(\frac{A}{B}) = \frac{P(A).P(B)}{P(A \cap B)}$

b) $P(\frac{A}{B}) = \frac{P(\frac{B}{A})P(A)}{P(B)}$

c) $P(\frac{A}{B}) = \frac{P(B)P(A)}{P(\frac{B}{A})}$

d) $P(\frac{A}{B}) = \frac{P(A \cap B)}{P(A)P(B)}$

(xiv) In the long run, the state probabilities become 0 & 1 _____. Select the correct option.

a) In no case

b) In some cases

c) In all cases

d) None of these

(xv) Markov analysis is used for _____. Select the correct option.

a) Predicting the state of the system at some future time

b) Calculating transition probabilities at some future time

c) Predicting the steadiness of the system at some future time

d) All of these

Group-B

(Short Answer Type Questions) 3 x 5=15

2. A bag contains 4 balls. Two balls are drawn at random without replacement and are found to be blue. Enumerate the probability that all balls in the bag are blue. (3)

3. Explain the probability mass function of Binomial distribution. (3)

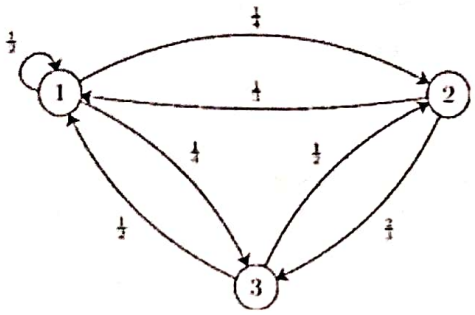
4. Determine the variance of geometric distribution. (3)

5. Given the following transition matrix for a Markov chain with states M, N, and O: (3)

$$P = \begin{pmatrix} 0.6 & 0.4 & 0 \\ 0.3 & 0.7 & 0 \\ 0.5 & 0 & 0.5 \end{pmatrix}$$

Classify each state (M, N, and O) as transient, recurrent, or absorbing. Justify your answer.

6. Write the TPM of the following Markov chain shown in Figure: (3)



OR

Consider the following transition matrix (3)

$$\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Justify if the matrix is valid TPM.

Group-C

(Long Answer Type Questions) 5 x 6=30

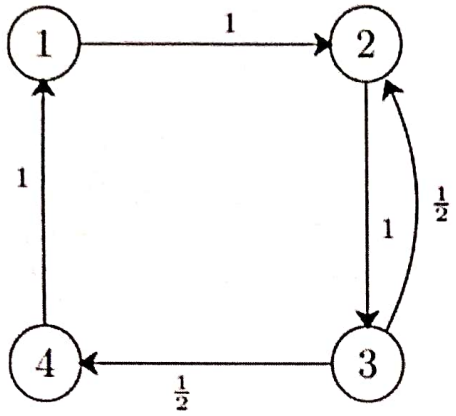
7. Consider the Markov chain with three states, S= {1,2,3}, that has the following transition matrix (5)

$$\begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

Test if this chain is communicative.

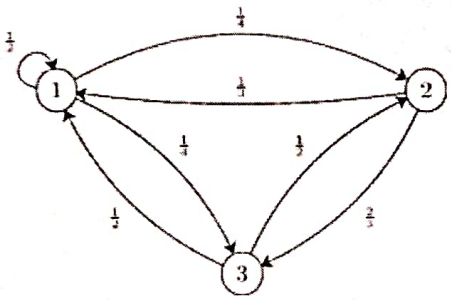
8. An HIV test gives a positive result with probability 98% when the patient is indeed affected by HIV, while it gives a negative result with 99% probability when the patient is not affected by HIV. If a patient is drawn at random from a population in which 0.1% of individuals are affected by HIV and he is found positive, identify the probability that he is indeed affected by HIV? (5)

9. Consider the Markov chain shown in Figure (5)



Test if this chain is aperiodic.

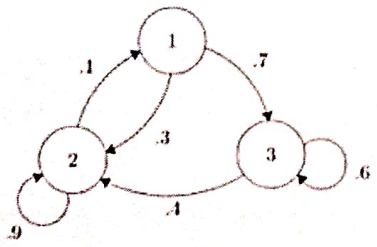
10. Consider the Markov chain shown in Figure (5)



Evaluate the stationary probabilities of the chain.

11. Compute mean and variance of Binomial distribution. (5)

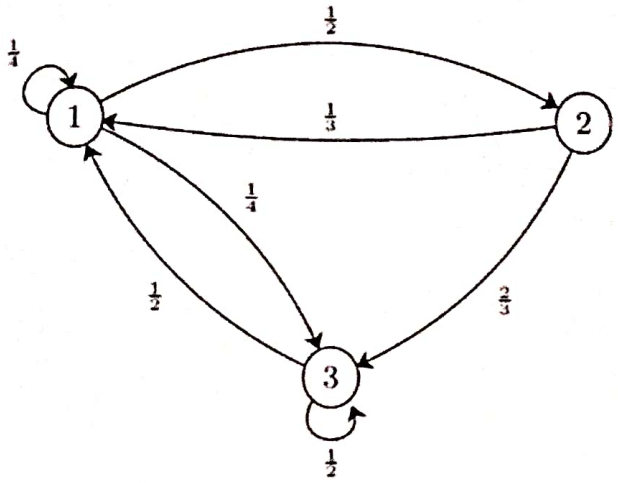
12. Consider the Markov chain shown in Figure (5)



Test if this chain is aperiodic.

OR

Consider the Markov chain shown in Figure (5)



Evaluate the stationary probabilities of the chain.

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