BRAINWARE UNIVERSITY

Term End Examination 2024-2025
Programme – M.Sc.(MATH)-2023
Course Name – Functional Analysis
Course Code - MSCMC301
(Semester III)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Select the correct answer. If set of vectors $\{(1,0,0),(1,x,1),(x,0,1)\}$ is linearly dependent, then x is

a) 0
b) 1
c) 2
d) 3

(ii) Select the correct answer. Let $\|\cdot\|$ be a norm on a vector space X. Then $\|x+y\|$

a) $=\|x\|+\|y\|$
b) $\geq\|x\|+\|y\|$
c) $\leq\|x\|+\|y\|$
d) None of these

(iii) Select the correct answer. Let $\|\cdot\|$ be a norm on a complex vector space X. Then $\|ix\|=$

a) $\|x\|$
b) $-\|x\|$
c) $\leq\|x\|$
d) $\geq\|x\|$

(iv) Select the correct answer. A normed linear space is a Banach space only if

a) converges imply absolute converges
b) absolute converges imply converges
c) converges imply absolute convergesabsolute converges imply converges"
d) None of these

(v) Select the correct answer. In a metric space X. A subset M is compact then

a) M is only closed
b) M is only bounded

c) M is both closed and bounded

d) M may not be closed or bounded

(vi) Select the correct answer. If in a normed space X has the property that the closed unit ball is compact then

a) X is finite dimensional

b) X is infinite dimensional

c) X is compact

d) None of these

(vii) Select the correct answer. Let $T : X \to Y$ be an unbounded linear operator between normed spaces X and Y. Then T is

a) always continuous

b) never continuous

c) Continuous at some points of X

d) T can't be unbounded

(viii) Write the correct answer. The set of all rational number Q in R

a) dense

b) nowhere dense

c) have countable interior points

d) other

(ix) Write the correct answer. To apply uniform boundedness theorem over the sequence of bounded linear operator $\{T_n\} \in B(X,Y)$. Then we consider X as a

a) only an NLS

b) only a complete space

c) Banach space

d) Other

(x) Write the correct answer. Let X be a vector space and dimension of X is n. Then the dimension of the algebraic dual X* of X is

a) $\leq n$

b) $\geq n$

c) n

d) Other

(xi) Select the correct answer.

a) Hilbert space is inner product space

b) All inner product spaces are Hilbert spaces

c) All Banach spaces are Hilbert spaces

d) Other

(xii) Select the correct answer. Let Y be a closed subspace of a Hilbert space X. Then Y is

a) compact

b) complete

c) convex but not complete

d) convex but not compact

(xiii) Select the correct answer. Let a Hilbert space H contains a finite family of total orthonormal set. Then Hilbert dimension of H is

a) countable

b) finite

c) uncountable

d) other

(xiv) Select the correct answer. Let M be a subset of an inner product space X and M is total in X. Then $x \perp M$ implies

a) $x \in M$

b) $x \notin M$

c) $x = 0$

d) None of these

(xv) Select the correct answer. Let H be a Hilbert space contains a total orthonormal sequence then H is

a) inseparable
b) separable
c) all dense set are uncountable
d) other

Group-B
(Short Answer Type Questions)

3 x 5=15

2. Analyze that $d(x,y)=\|x-y\|, x,y\in X$ is a metric on the NLS $(X,\|.\|)$. (3)

3. Give an example of a Banach space which is not a Hilbert space. (3)

4. Explain that L(X, Y) is a linear space, where X and Y are NLS's. (3)

5. State polarization identity. (3)

6. Justify that the space $l^p, p \neq 2$ is not a Hilbert space. (3)

OR

Justify that the inverse operator T^{-1} of a linear operator of T is a linear operator. (3)

Group-C
(Long Answer Type Questions)

5 x 6=30

7. Show that the set S={(0,1,1),(1,0,1).(1,1,1)} is a basis for R^3 (5)

8. Explain the following statement. (5)
For every linear operator $T: X \to Y$ we have

$$T\left(\sum_{i=1}^{n} \alpha_i x_i\right) = \sum_{i=1}^{n} \alpha_i T x_i$$

9. Explain that dual space of $\mathbb{R}^n$ is $\mathbb{R}^n$. (5)

10. Explain that for every non-zero x in a normed linear space X, (5)
$\|x\| = \sup_{f\in X^*, f\neq 0} \frac{|f(x)|}{\|f\|}$.

11. From Schwarz inequality justify that norm satisfies the triangle inequality. (5)

12. Let M be a complement subspace Y and $x \in X$ fixed. Then justify that $z = x - y$ is orthogonal to Y. (5)

OR

Justify that a linear subspace M of a Hilbert space H is closed in H if and only if $M = M^{\perp\perp}$. (5)

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