BRAINWARE UNIVERSITY

Term End Examination 2024-2025
Programme – M.Sc.(MATH)-2023
Course Name – Differential Geometry
Course Code - MSCMC302
(Semester III)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Identify the rectangular Cartesian coordinates of the point whose cylindrical coordinates are $\left(2,\frac{\pi}{3},1\right)$

a) $\left(1,\sqrt{3},1\right)$
b) $(1,2,1)$
c) $(1,3,1)$
d) None of these

(ii) Identify the necessary condition for a function to be considered differentiable at a specific point

a) The function must be continuous at that point.
b) The function must have a local maximum or minimum at that point.
c) The function must be defined for all real numbers.
d) The function must have a vertical asymptote at that point.

(iii) Identify the type of point is a differentiable curve that guaranteed to have a horizontal tangent.

a) Critical point
b) Inflection point
c) Maximum point
d) Discontinuity point

(iv) Select the correct option. Differentiable functions on surfaces are mappings that:

a) Map one surface to another
b) Assign real numbers to points on the surface
c) Map surfaces to lines
d) Map surfaces to curves

(v) Select the correct option. Normal curvature of a surface at a point is the curvature of the surface:

a) In the direction of the tangent vector at that point
b) In the direction of the normal vector at that point
c) In the direction of the binormal vector at that point
d) In the direction of the surface's axis of symmetry

(vi) Select the correct option. Gaussian curvature is the product of the principal curvatures of a surface. It is:

a) Always positive
b) Always negative
c) Always zero
d) Can be positive, negative, or zero depending on the surface's shape

(vii) Evaluate the relationship between the normal curvature and the principal curvature in the direction of the normal vector.

a) They are equal
b) They are opposite in sign
c) They are perpendicular to each other
d) They are parallel to each other

(viii) Select the correct option. The Jacobi vector field describes the deviation of a geodesic due to changes in the:

a) Principal curvature
b) Gaussian curvature
c) Exponential map
d) Second fundamental form

(ix) If θ be the angle between two parametric curves, then evaluate $\tan\theta =$

a) $\dfrac{\sqrt{a}}{a_{12}}$
b) $\dfrac{\sqrt{a}}{a_{22}a_{11}}$
c) $\dfrac{\sqrt{a}}{a_{11}}$
d) None of these

(x) Judge the correct option: δ^i_i is

a) 1
b) An invariant
c) Not an invariant
d) None of these

(xi) Determine the kronecker delta δ^i_j

a) Symmetric in i and j
b) Skew-symmetric in i and j
c) 1
d) None of these

(xii) If A_{ij} is a skew-symmetric tensor, identify $\left(\delta^i_j\delta^k_i - \delta^i_i\delta^k_j\right)A_{ik} =$

a) 1
b) 0
c) n
d) None of these

(xiii) The coefficient g_{ij} defined in $ds^2 = g_{mn}dx^m dx^n$ then identify the components

a) Skew-symmetric covariant tensor of type (0,2)
b) Symmetric covariant tensor of type (0,2)
c) Skew-symmetric covariant tensor of type (1,2)
d) Symmetric covariant tensor of type (1,2)

(xiv) Determine the angle between two non-null vectors

a) Invariant
b) Not invariant
c) 0
d) None of these

(xv) If $g_{ij} = 0$ for $i \neq j$, then $[i\,j, i] =$ ________________ (Identify the correct option)

a) $\dfrac{1}{2}\cdot\dfrac{\partial g_{ii}}{\partial x^i}$
b) $-\dfrac{1}{2g_{ii}}\cdot\dfrac{\partial g_{jj}}{\partial x^i}$
c) $-\dfrac{1}{2}\cdot\dfrac{\partial g_{ii}}{\partial x^k}$
d) $\dfrac{1}{2}\cdot\dfrac{\partial g_{ii}}{\partial x^j}$

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Evaluate the equation of the tangent to the curve $y^2 - yx^2 - 2x^5 = 0$ at the point (1,-1). (3)

3. Define contravariant tensor. (3)

4. In terms of Cartesian parametric representation, show that $s = \int_a^u \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\, du$. (3)

5. Write the definition of arc-length of a curve. (3)

6. Write the first fundamental form. (3)

OR

Write the Frenet-Serret formulae of a smooth curve in 3-dimensinal Euclidean space. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Describe what do you mean by curvature and radius of curvature of a curve. (5)

8. Evaluate the indicatrix of the binormal of the helix γ given by r = (a cos u, a sin u, bu). (5)

9. Define principal normal and binormal. (5)

10. Evaluate the length of the normal for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}})$. (5)

11. If $B_{ij} = A_{ji}$, where A_{ij} is a covariant tensor, then justify that B_{ij} is a tensor of order 2. (5)

12. If $a_{ij}v^i v^j = b_{ij}v^i v^j$ for an arbitrary contravariant vector v^i, then justify that $a_{ij} + a_{ji} = b_{ij} + b_{ji}$. (5)

OR

If $a_{ij}b^{ik} = c_j^k$, then justify that $(b^{ik})^T (a_{ij}) = (c_j^k)$ and $|b^{ik}||a_{ij}| = |c_j^k|$. (5)

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