BRAINWARE UNIVERSITY

Term End Examination 2024-2025
Programme – M.Sc.(MATH)-2023
Course Name – Integral Equations & Calculus of Variations
Course Code - MSCMC303
(Semester III)

Full Marks : 60

Time : 2:30 Hours

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Select the correct option. An integral equation is an equation in which the unknown function appears under one or more integral signs.

a) always true
b) always false
c) maybe true
d) None of these

(ii) Select the correct option. The function $K(x,t)$ of $g(x)y(x) = f(x) + \lambda \int_a K(x,t)\, y(t)dt$ is called ________ of the integral equation.

a) Kernel
b) Integral
c) integral constant
d) None of these

(iii) If g(x)=0, in $g(x)y(x) = f(x) + \lambda \int_a K(x,t)\, y(t)dt$, then the equation can be identified as

a) linear integral equation of 1st kind
b) linear integral equation of 2nd kind
c) linear integral equation of 3rd kind
d) None of these.

(iv) For the kernel $K(x,t)$, if $K(x,t) = K(x-t)$, then $K(x,t)$ can be identified as

a) Separable Kernel
b) Degenerate Kernel
c) Difference Kernel
d) None of these.

(v) The integral equation obtained from

$y" - xy' + y = 0 \; with \; y(0) = 1, y'(0) = -1$ can be expressed as

a) Volterra equation of 1st kind
b) Fredholm equation of 1st kind

c) Volterra equation of 2nd kind d) Fredholm equation of 2nd kind

(vi) Choose the correct option: The IVP corresponding to the integral equation $y(x) = 1 + \int_0^x y(t)dt$ is

a) $y' - y = 0, y(0) = 1$
b) $y' + y = 0, y(0) = 0$
c) $y' - y = 0, y(0) = 0$
d) $y' + y = 0, y(0) = 1$

(vii) The integral equation obtained from $y'' - sinxy' + e^x y = 0$ $with$ $y(0) = 1, y'(0) = -1$ can be classified as

a) Volterra equation of 1st kind
b) Fredholm equation of 1st kind
c) Volterra equation of 2nd kind
d) Fredholm equation of 2nd kind

(viii) Calculate the value of λ for which the solution of the equation

$y(x) = cosx + \lambda \int_0^{\pi} sinx\ y(t)dt$ is $y(x) = cosx$,

a) $\lambda \neq 1$
b) $\lambda \neq 2$
c) $\lambda \neq \frac{1}{2}$
d) None of these.

(ix) Select the correct option. A mapping $f: X \to X$ is a contraction mapping if there exists a constant k such that

a) $d(x, y) = kd(f(x), f(y))$
b) $d(x, y) \leq kd(f(x), f(y))$
c) $d(x, y) \geq kd(f(x), f(y))$
d) Other

(x) Evaluate the solution of Fredholm integral equation $y(x) + \int_0^1 e^{x-t} y(t)dt = 2xe^x$

a) $y(x) = e^x(1 - x^2)$
b) $y(x) = e^x\left(2x - \frac{2}{3}\right)$
c) $y(x) = e^x\left(x - \frac{2x}{3}\right)$
d) None of these

(xi) Determine the Laplace transformation of cos (at)

a) $\frac{a}{a^2+s^2}$
b) $\frac{a^2}{a^2+s^2}$
c) $\frac{s}{a^2+s^2}$
d) $\frac{as}{a^2+s^2}$

(xii) Illustrate the solution of $y(x) = 2x - \pi + 4\int_0^{\frac{\pi}{2}} sin^2 x\, y(t)dt$

a) $y(x) = 2x - \pi + \frac{\pi^2 sin^2 x}{\pi - 1}$
b) $y(x) = 2x - \pi + \frac{\pi^2 cos^2 x}{\pi - 1}$
c) $y(x) = 2x - \pi + \frac{\pi^2 tan^2 x}{\pi - 1}$
d) None of these

(xiii) Select the correct option. $f(x) = \sin x$ is a periodic function of period

a) $\frac{\pi}{2}$
b) π

c) 2π

d) Other

(xiv) Select the correct option. If a functional I[y(x)] having a variation attains a .... on $y = y_0(x)$ then at $y = y_0(x)$, $\delta(I) = 0$

a) maximum

b) minimum

c) maximum or minimum

d) none of these.

(xv) Justify that the geodesics on a plane is a

a) circle

b) parabola

c) straight line

d) None of these.

Group-B

(Short Answer Type Questions) 3 x 5=15

2. State and prove the linearity property of the Fourier transformation. (3)

3. Define Fredholm integral equation with an Example. (3)

4. Solve the Volterra equation (3)

$$y(x) = 1 + \int_0^x x\,t\,y(t)dt$$

5. Illustrate the solution of the integral equation (3)

$$\int_0^x \frac{f(t)}{(x-t)^{1/3}}dt = x(1+x)$$

6. Classify Brachistochrone problem. (3)

OR

Deduce the Euler's equation for the extremals of the functional (3)

$$\int_{x_1}^{x_2} \{5y\text{^}3 - 15y' + 2y'^2\}\,dx$$

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Solve the following integral equation (5)

$$y(x) = e^x + \lambda \int_0^1 2e^x e^t y(t)dt$$

8. Illustrate the derivative of the following integration (5)

$$I(t) = \int_0^t x^2 e^x \, dx$$

9. Explain Continuity of functional and Linear functional. (5)

10. If $\alpha(x)$ is continuous in $[a, b]$ and if $\int_a^b \alpha(x) h(x) dx = 0$ for every function $h(x) \in C[a, b]$ such that $h(a) = 0 = h(b)$, then deduce that $\alpha(x) = 0$ for all $x \in [a, b]$. (5)

11. Deduce the stationary function of the functional (5)
$\int_a^b \{y'^2 + y\, y''\}\, dx : y(a) = \lambda_1, y'(a) = \lambda_2, y(b) = \lambda_3, y'(b) = \lambda_4$.

12. Evaluate the extremes of the functional (5)
$\int_0^{\frac{\pi}{4}} (y''^2 - y^2 + x^2)\, dx : y(0) = 0, y'(0) = 1, y\left(\frac{\pi}{4}\right) = y'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$.

OR

Choose the curve on which functional $\int_0^1 [(y')^2 + 2y] dx$ with boundary conditions $y(0) = 1$ and $y(1) = 0$ can be extremized, if exists. (5)

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