BRAINWARE UNIVERSITY

Term End Examination 2024-2025
Programme – M.Sc.(MATH)-2023
Course Name – Analytical Number Theory
Course Code - MSCME301C
(Semester III)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) In Euler phi (totient) function, it can be observed that
a) $m = n$ b) $m \geq n$
c) $m \leq n$ d) None of these

(ii) From definition of Moebius function, we can recognize
a) Absolute value of μ(n) b) The square of μ(n)
c) Characteristic function of squarefree integer d) All of these

(iii) In big oh, the functions f(x) and g(x) can be expressed for all
a) Sufficiently large x b) Sufficiently small x
c) For a fixed x d) All of these

(iv) The Prime Number Theorem describes about the distribution of prime numbers that
a) Prime numbers become less frequent as numbers increase. b) Prime numbers become more frequent as numbers increase.
c) Prime numbers are evenly distributed throughout all positive integers d) There is no pattern in the distribution of prime numbers

(v) Identify the correct option. The value of the partial sum $\sum\varphi(n)$ for all positive integers n less than or equal to 5 is
a) 5 b) 6
c) 8 d) 10

(vi) The Mellin transform of a constant function can be generalized as
a) A delta function b) Zero
c) Infinity d) The original constant function

(vii) Choose the right statement.
a) The Euler product representation is valid for all Dirichlet series. b) The Euler product representation is a general property of all Dirichlet series.

c) The Euler product representation holds only for certain classes of Dirichlet series.

d) The Euler product representation is equivalent to the Dirichlet series representation.

(viii) Choose the correct statement regarding the Möbius inversion formula.

a) The Möbius inversion formula relates the Möbius function to the prime-counting function.

b) The Möbius inversion formula expresses a function in terms of its Dirichlet convolution with the Möbius function.

c) The Möbius inversion formula is only applicable to even numbers.

d) The Möbius inversion formula gives a direct formula for calculating the Möbius function.

(ix) Select the correct statement about the Kronecker's Lemma.

a) It is a fundamental result in algebraic number theory.

b) It provides a proof for Fermat's Last Theorem.

c) It guarantees the existence of infinitely many prime numbers.

d) It establishes a connection between complex analysis and number theory.

(x) Choose the condition for the Dirichlet series associated with the Dirichlet character $\chi(n)$ is convergent for:

a) Re(s) > 0

b) Re(s) > -1

c) Re(s) > 1

d) All complex numbers s

(xi) Select the correct option: The Riemann zeta function satisfies the functional equation

a) $\zeta(s) = \zeta(1 - s)$

b) $\zeta(s) = \zeta(s\ 1)$

c) $\zeta(s) = \zeta(2s)$

d) $\zeta(s) = \zeta(2 - s)$

(xii) The Chebyshev prime number theorem gives an estimate for

a) The number of twin primes up to a given bound.

b) The number of composite numbers up to a given bound.

c) The distribution of prime numbers up to a given bound.

d) The sum of all prime numbers up to a given bound.

(xiii) Conclude the lower bounds for error terms in summatory functions typically used for

a) Approximating the exact summatory function

b) Estimating the upper bound of the error.

c) Estimating the lower bound of the error.

d) Finding the mean of the error.

(xiv) The Hadamard product formula for the Riemann zeta function provides a simple expression for the non-trivial zeros then select the form

a) $\zeta(s) = \prod(s - \rho)$

b) $\zeta(s) = \prod(1 - \frac{s}{\rho})$

c) $\zeta(s) = \prod(1 - \frac{\rho}{s})$

d) $\zeta(s) = \prod(s + \rho)$

(xv) Select the value of constant in Stirling's formula

a) $\sqrt{\pi}$

b) $\sqrt{2\pi}$

c) $\sqrt{3\pi}$

d) None of these

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Explain Euler totient phi function. (3)

3. Explain Kronecker's Lemma (3)

4. Illustrate analytic properties of Dirichlet series. (3)

5. Examine Moebius function using Dirichlet series. (3)

6. Summarize Euler product identity. (3)

OR

Summarize the logarithmic integral formula. (3)

Group-C
(Long Answer Type Questions) 5 x 6=30

7. Show that for $n \geq 1$, $\sum[d|n]\ \mu(d)=0$. (5)

8. State and prove Euler's summation formula. (5)

9. Evaluate Wintner's mean value theorem. (5)
10. Evaluate Mertens constant. (5)
11. Deduce the Characterization of multiplicative functions (5)
12. Evaluate an application of Perron's formula. (5)

OR

Justify Stirling's formula. (5)

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