BRAINWARE UNIVERSITY

Term End Examination 2024-2025
Programme – M.Sc.(MATH)-2023
Course Name – Discrete Structures
Course Code - MSCME302D
(Semester III)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Express the generating function for generating series 1, 2, 3, 4, 5,...

a) $2/(1-3x)$ b) $1/(1+x)$
c) $1/(1-x)^2$ d) $1/(1-x^2)$

(ii) Suppose X = {a, b, c, d} and π_1 is the partition of X, π_1 = {{a, b, c}, d}. Calculate the number of ordered pairs of the equivalence relations induced by the partition.

a) 15 b) 10
c) 34 d) 5

(iii) Consider the ordering relation a | b ⊆ N x N over natural numbers N such that a | b if there exists c belong to N such that a*c=b. Then select the correct option from following.

a) | is an equivalence relation b) It is a total order
c) Every subset of N has an upper bound under | d) (N, |) is a lattice but not a complete lattice

(iv) The total number of ways you can select 5 letters from the letters of the word INDEPENDENT is

a) 72 b) 27
c) 462 d) None of these

(v) Let A = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16} and consider the divides relation on A. Let C denote the length of the maximal chain, M the number of maximal elements, and m the number of minimal elements. Tell the true statement.

a) C = 3, M = 8, m = 6 b) C = 4, M = 8, m = 6
c) C = 3, M = 6, m = 6 d) C = 4, M = 6, m = 4

(vi) Calculate the number of numbers which are chosen from 1 to 7 so that two of them will add up to 8.

a) 3 b) 4
c) 5 d) 9

(vii) For the sequence 1, 7, 25, 79, 241, 727 ... construct the simple formula for n^{th} term.

a) $3^{n+1}-2$ b) 3^n-2
c) $(-3)^n+4$ d) n^2-2

(viii) Calculate the value of a_4 for the recurrence relation $a_n=2a_{n-1}+3$, with $a_0=6$.

a) 320 b) 124

c) 141 d) 56

(ix) If set A is empty set then compute $n\left[P\left[P\left[P(A)\right]\right]\right]=$

a) 6 b) 16

c) 2 d) 4

(x) A survey shows that 70% of Indians like mango whereas 82% like apples. If x% of Indian like both mango and apples then evaluate x.

a) $x=52$ b) $52\le x\le 70$

c) $x=70$ d) $70\le x\le 82$

(xi) Let $D_{30}=\{1, 2, 3, 5, 6, 10, 15, 30\}$ and relation I be a partial ordering on D_{30}. Evaluate the lub of 10 and 15 respectively.

a) 30 b) 15

c) 10 d) 6

(xii) Choose the correct option. Prim's algorithm is used for:

a) Finding the shortest path between two vertices b) Finding the minimum spanning tree

c) Finding Hamiltonian paths d) Testing graph isomorphism

(xiii) Select the multiplication of the sequence 1, 2, 3, 4,... by the sequence 1, 3, 5, 7, 11,....

a) 1, 5, 14, 30,... b) 2, 8, 16, 35,...

c) 1, 4, 7, 9, 13,... d) 4, 8, 9, 14, 28,...

(xiv) Select the sequence generated by the generating function $4x/(1-x)^2$

a) 12, 16, 20, 24,... b) 1, 3, 5, 7, 9,...

c) 0, 4, 8, 12, 16, 20,... d) 0, 1, 1, 3, 5, 8, 13,...

(xv) Define a spanning tree

a) A subgraph containing all the vertices and some of the edges of the graph b) A connected acyclic subgraph that includes all vertices of the original graph

c) A tree where all vertices have the same degree d) A graph with exactly one cycle

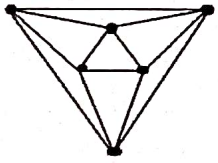
Group-B

(Short Answer Type Questions) 3 x 5=15

2. Define a bounded lattice. (3)

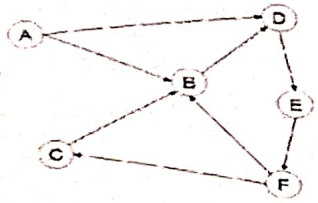
3. Show that $a_n=2n+1$ is a solution to the recurrence relation $a_n=2a_{n-1}-1$ with $a_1=3$. (3)

4. Construct the dual of the following graph: (3)



5. Explain the complement of an element in a lattice. (3)

6. Evaluate the adjacency and incidence matrix of the following undirected graph. (3)



OR

Let $S = \{1,2,3,4,6,9,12,18,36\}$. Evaluate the Infimum and Supremum of the sets $\{6,18\}$ and $\{4,6,9\}$ in the Poset$(S, /)$ (3)

Group-C
(Long Answer Type Questions) 5 x 6=30

7. Explain the difference between a path and a cycle in a graph. (5)

8. Evaluate the minimum spanning tree and its usefulness. (5)

9. Express the generating functions for the following sequences. In each case, try to simplify the answer. (5)
(i) 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, ...
(ii) 1, 1, 1, 1, 1, ...
(iii) 1, 3, 3, 1, 0, 0, 0, 0, ...

10. Illustrate the process of constructing a Hasse diagram for a poset. (5)

11. Explain the concept of lattice as a poset with an example. (5)

12. Explain Euler's Polyhedron formula and justify its significance in planar graphs. (5)

OR

Select the correct one Poset represented by the following Hasse diagram is Lattice and give proper reason. (5)