BRAINWARE UNIVERSITY

Term End Examination 2024-2025
Programme – B.Tech.(CSE)-AIML-2023/B.Tech.(CSE)-DS-2023
Course Name – Differential Equation and Complex Analysis
Course Code - BSCM301/BSCD301
(Semester III)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Select the correct option: Let γ be the curve $g(x,y) = x^3y + xy^3 = 5$. Then the value of $\int_\gamma \vec{\nabla} g.dr$

a) 2 b) 4

c) 0 d) None of the mentioned

(ii) Select the correct option.: If $i + j + k, 2i - 4k, i + \lambda j + 3k$ are coplanar, then λ is

a) 2 b) 4/3

c) 5/3 d) None of these

(iii) Select the vector field from the following that is conservative

a) $F(x,y)=(xe^x+y)\hat{i}+x\hat{j}$ b) $F(x,y)=(xe^x+y)\hat{i}+2x\hat{j}$

c) $F(x,y)=(xe^x+2y)\hat{i}+x\hat{j}$ d) None of the mentioned

(iv) Let $F(x,y) = (ax^2y + y^3 + 1)\hat{i} + (2x^3 + bxy^2 + 2)\hat{j}$ be a vector field, where a and b are constants. Select the values of a and b respectively such that the vector field F becomes conservative.

a) 3, 6 b) 6, 3

c) 1, 2 d) 2, 1

(v) Select the flux of $F = x\hat{i} + y\hat{j} + z\hat{k}$ through the sphere of radius a and center at the origin. Take $\hat{n}$ pointing outward.

a) 4π b) $4\pi a^2$

c) $4\pi a^3$ d) None of these

(vi) Let z_0 be a point in the z plane. If $\lim_{z\to z_0} \frac{1}{f(z)} = 0$, then write $\lim_{z\to z_0} f(z) =$

a) 0 b) ∞

c) Does not exist d) None of these

(vii) Write the value of the limit $\lim_{z\to(-1)} \frac{iz+3}{z+1}$

a) 0 b) ∞

c) Does not exist d) None of the mentioned

(viii) Write the correct one: The function $f(z) = \begin{cases} z^2, & z \neq 1 \\ 0, & z = 1 \end{cases}$ is discontinuous at

a) 0 b) 1

c) everywhere in the complex plane d) nowhere in the complex plane

(ix) Choose the correct option: If the real part of an analytic function be $u = x^2 - y^2$, then the analytic function is

a) $iz^2 + c$ b) $-iz^2 + c$

c) $z + c$ d) $z^2 + c$

(x) Choose the correct option: If $f(z)$ is analytic and non-zero in a domain D, then in D, $\log|f(z)|$ is

a) Analytic b) A constant

c) Zero d) Harmonic

(xi) Identify the correct answer. If a function is analytic at all points of a bounded domain except at finitely many points, then these points are called:

a) Zeros b) Singularities

c) Poles d) Simple points

(xii) Identify the type of singularity for the function $f(z) = e^z$ at $z = \infty$.

a) isolated essential singularity b) pole

c) ordinary point d) None of these

(xiii) Identify the correct one: $\frac{2!}{2\pi i}\int_{|z|=3} \frac{z^2+3z+4}{(z-1)^3} dz =$

a) 2 b) 0

c) πi d) None of these

(xiv) For any closed curve γ and $a \in \gamma$, then identify the index (or, winding) number $\eta(\gamma; a)$

a) an integer b) a rational number

c) an irrational number d) a fraction

(xv) For $|z| < 2$, identify the series expansion of the function $f(z) = \frac{1}{z-2}$

a) $\sum_{n=0}^{\infty} \frac{z^n}{2^{n+1}}$

b) $-\sum_{n=0}^{\infty} \frac{z^n}{2^{n+1}}$

c) $\sum_{n=0}^{\infty} \frac{z^n}{2^n}$

d) $\sum_{n=0}^{\infty} \frac{z^{n+1}}{2^{n+1}}$

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Define irrational vector field and solenoidal vector field with example. (3)

3. Calculate the Taylor series representation of $f(z) = 1/(z^2-3z+2)$ about the origin. (3)

4. Illustrate the concept of harmonic conjugate of a harmonic function and describe it with an example. (3)

5. Solve the differential equation $(4D^2 + 4D - 3)y = e^{2x}$ (3)

6. Evaluate the value of the following integral $\int_C \frac{\sin z}{(z-2)(z-3)} dz$, C is the circle $|z| = 2.5$ (3)

OR

Conclude Cauchy's Residue theorem. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Identify the residues of the function $\frac{z+1}{z^2(z-3)}$ at its singularities. (5)

8. Evaluate $\int_C \frac{z^3}{z^2+5z+6} dz$, given that C is the circle $|z| = 1$, positively oriented. (5)

9. Calculate the solution of the differential equation $y'' + y = \sec x$ using the method of variation of parameters. (5)

10. Illustrate if the functions $z\bar{z}$ and z^2 are analytic. If so, then determine the domain of analyticity. (5)

11. Use Stokes theorem to recognize $\int_c \vec{F} \cdot d\vec{r}$, $\vec{F} = z^2\hat{\imath} + y^2\hat{\jmath} + x\hat{k}$ and C is the triangle with vertices (1,0,0), (0,1,0) and (0,0,1) with counter-clockwise rotation. (5)

12. Solve the integral $\oint \frac{z^2\,dz}{z^2-1}$ around the unit circle with center at (a) $z = 1$, (b) $z = -1$. (5)

OR

Solve the integrals $\oint \frac{z^2+1}{(z-1)^2}dz$ and $\oint \frac{\cos z}{z^3}dz$ around the circle $|z| = 2$. (5)

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