BRAINWARE UNIVERSITY

Term End Examination 2024-2025
Programme – B.Tech.(RA)-2022
Course Name – Probability Theory and Stochastic Process
Course Code - BSCR501
(Semester V)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Let X be a random variable with mean μ and variance σ^2 and k be any real number. Choose the statement(s) that represents Chebychev's inequality.

a) $P\{|X-\mu| \geq k\sigma\} \leq \frac{1}{k^2}$

b) $P\{|X-\mu| < k\,\sigma\} \geq 1-\frac{1}{k^2}$

c) $P\{|X-\mu| \geq k\sigma\} \leq \frac{1}{k^2}$ and $P\{|X-\mu| < k\,\sigma\} \geq 1-\frac{1}{k^2}$

d) None of these

(ii) Let X be a random variable with mean μ and variance σ^2 and k be any real number. Choose the statement that represents Markov's inequality.

a) $P\{|X-\mu| \geq k\sigma\} \leq \frac{1}{k^2}$

b) $P\{|X-\mu| < k\,\sigma\} \geq 1-\frac{1}{k^2}$

c) $P\{|X| \geq k\} \leq \frac{E|X|}{k}$

d) None of these

(iii) Sum of total probabilities in a sample space is _______. Select the correct option.

a) 1/2

b) 1

c) 0

d) Cannot tell

(iv) Two unbiased coins are tossed. Then compute the probability of obtaining at least one tail.

a) 3/4
b) 1/2
c) 1/4
d) None of these

(v) A set of all possible outcomes of an experiment is called _______. Select the correct option.

a) Combination
b) Sample point
c) Sample space
d) Compound event

(vi) Compute the probability of getting at least one of getting a 'six' or 'one' on the top in rolling of an unbiased die once.

a) $\frac{1}{6}$
b) $\frac{1}{9}$
c) $\frac{1}{3}$
d) None of these

(vii) The second central moment is _____. Choose the correct option.

a) Mean
b) Median
c) Range
d) Variance

(viii) A stationary random process X(t) is periodic with period 2T. Its auto correlation function is _________. Select the correct option.

a) Non-periodic
b) periodic with period T
c) Periodic with period 2T
d) periodic with period T/2

(ix) If $X_1, X_2, \ldots$ are independent and identically distributed with mean μ, then compute $P\{\lim_{n\to\infty} (X_1 + X_2 + \cdots + X_n)/n = \mu\}$

a) 0
b) 1
c) 2
d) None of these

(x) Let $\{Z_n\}_{n\in N_0}$ be a branching process with offspring distribution $(p_0, p_1, \ldots)$. We have $P(s) = \sum_{n=0}^{\infty} p_n s^n$. The extinction is inevitable if _____. Select the correct option.

a) $p_0 > 0$
b) $p\left(\frac{1}{2}\right) > \frac{1}{4}$
c) $p'(1) < 1$
d) $p'(1) > 1$

(xi) An absorbing state of a Markov chain is one in which the probability of _________. Select the correct option.

a) moving into that state is zero.
b) moving out of that state is one.
c) moving into that state is one.
d) moving out of that state is zero.

(xii) The steady-state probability vector π of a discrete Markov chain with transition probability matrix P satisfies the matrix equation of __________. Select the correct option.

a) $P\pi = 0$
b) $(2-P)\pi = 0$
c) $P\,\pi = \pi$
d) $P^t\pi = 0$

(xiii) If a matrix of transition probability is of order $n \times n$, then select the number of equilibrium equations.

a) n
b) n-1
c) n+1
d) n+2

(xiv) Let $\{X_n\}, n = 1,2,3 \ldots$ be a Markov Chain with k states. A state $j, j = 1,2, \ldots k$ is said to be ______ state if the return of the chain to state j for the first time after n steps, having started from the same state j, is uncertain. Select the correct option.

a) Accessible
b) Persistent
c) Absorbing
d) Transient

(xv) Let $\pi_i^{(n)}$ represent the probability that the Markov chain with k states is in state i at step n. Then _____. Select the correct option.

a) $\sum_{i=1}^{k} \pi_i^{(n)} = 1.$
b) $\sum_{i=1}^{k} \pi_i^{(n)} = 0.$
c) $\sum_{i=1}^{k} \pi_i^{(n)} < 1.$
d) None of these

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Explain Chebychev's inequality. (3)

3. Consider a Markov chain with two states 1, 2. Suppose that $p_{1,2} = a,\ p_{2,1} = b$. Compute the values of a and b for which we obtain an absorbing Markov chain. (3)

4. Explain Poisson Process and Renewal Process. (3)

5. The transition probability matrix of a Markov Chain with 3 states 1, 2, 3 is given as (3)

$$P = \begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.4 & 0 & 0.6 \\ 0 & 0.8 & 0.2 \end{pmatrix}$$

Illustrate the state transition diagram.

6. Summarize M/M/1, M/M/c, and M/G/1 Queuing systems. (3)

OR

Explain the Kolmogorov's forward and backward equations for Continuous-Time Markov Chains. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Summarize the concepts of merging independent Bernoulli processes. (5)

8. Explain Probability mass function with an example. (5)

9. Evaluate the steady-state probabilities of the Markov Chain having one-step TPM (5)

$$P^{(1)} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \end{pmatrix}.$$

10. An economics consulting firm has created a model to predict recessions. The model predicts a recession with probability 80% when a recession is indeed coming and with probability 10% when no recession is coming. The unconditional probability of falling into a recession is 20%. If the model predicts a recession, compute the probability that a recession will indeed come. (5)

11. Explain the probability density function of normal distribution. (5)

12. The number of orders arriving at a service facility can be modelled by a Poisson process with intensity λ=10 orders per hour. Evaluate the probability that there are no orders between 10:30 and 11. (5)

OR

Evaluate the limiting probability distribution of the Markov Chain having one-step Transition Probability Matrix $P = \begin{pmatrix} 1-a & a \\ b & 1-b \end{pmatrix}$. (5)

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