14499

BRAINWARE UNIVERSITY

Term End Examination 2024-2025
Programme – M.Sc.(MATH)-2024
Course Name – Abstract Algebra
Course Code - MSCMC201
(Semester II)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) The number of generators of the cyclic group $(\mathbb{Z}_{10}, +)$ is

[redacted]

a) 3 b) 4
c) 2 d) 5

(ii) Choose the correct option. If H and K be subgroups of a group G with $O(H) = 8$, $O(K) = 12$ and $O(HK) = 24$, then $O(H \cap K)$ is

a) 2 b) 8
c) 12 d) 4

(iii) Choose the correct option. If a be an element of G of order n and p is prime to n, then $O(a^p)$ is

a) 1 b) n
c) np d) None of these

(iv) Identify the correct option. In Klein's 4-group $\{e, a, b, c\}$, the order of each non-identity elements is

a) 1 b) 2
c) 3 d) 4

(v) Write the cardinality of the center of Z_{12}

[redacted]

a) 1
b) 2
c) 3
d) 12

(vi) Select the correct option. Let G be a group and H be a subgroup of G and $h \in H$, then

a) $H \subset hH$
b) $H \neq hH$
c) $hH = H$
d) None of these

(vii) If a group is abelian, then choose the correct option from below that is true for its quotient group.

a) Non-abelian
b) Abelian
c) Complex
d) Cyclic

(viii) If $G = \{1, -1, i, -i\}$ is a multiplicative group, then identify the order of $-i$

a) 1
b) 2
c) 3
d) 4

(ix) Select the correct option. If integral domain I is of finite characteristic, then

a) I is finite only
b) I is infinite only
c) I is finite or infinite only
d) None of these

(x) Identify the correct statement

a) The polynomial $x^2 - 2$ is irreducible over the field of rational numbers while it is irreducible over the field of real numbers.
b) The polynomial $x^2 - 2$ is irreducible over the field of rational numbers. While it is reducible over the field of real numbers.
c) The polynomial $x^2 + 1$ is reducible over the field of real numbers.
d) The polynomial $x^2 + 1$ is irreducible over the field of complex numbers.

(xi) Select the correct option. The characteristic of the ring of even integers is

a) 2
b) 1
c) 0
d) None of these

(xii) Identify the correct option from the following that is not a field.

a) $\mathbb{Z}/2\mathbb{Z}$
b) $\mathbb{Z}/4\mathbb{Z}$
c) $\mathbb{Z}/11\mathbb{Z}$
d) $\mathbb{Z}/5\mathbb{Z}$

(xiii) Write the correct option. K/F is said to be simple extension if

a) $F = K(\alpha)$
b) $K = F(\alpha)$
c) $K = F$
d) None of these

(xiv) Write the correct option. Degree of $\mathbb{Q}(\sqrt[5]{2}, \sqrt{3})/\mathbb{Q}$ is

a) 10
b) 6
c) 15
d) 5

(xv) Select the correct option. If an abelian group G of order 10 is cyclic, then G contains an element of order:

a) 2 b) 4
c) 6 d) 5

Group-B
(Short Answer Type Questions) 3 x 5=15

2. Show that the characteristic of an integral domain is either zero or a prime number. (3)

3. Illustrate the definition of normal extension and give an example. (3)

[redacted]

4. Define simple group. Show that every group of prime order is simple. (3)

5. Show that a group of order 35 is cyclic. (3)

6. Justify that $SL(n, \mathbb{R})$ is a normal subgroup of $GL(n, \mathbb{R})$. (3)

OR

Let G be a group and $Z(G)$ be the centre of G. Then justify that if $a \in Z(G)$ then $cl(a) = \{a\}$ and conversely. (3)

Group-C
(Long Answer Type Questions) 5 x 6=30

7. Let G be a non-commutative group of order p^3, p a prime. Evaluate $|Z(G)|$. (5)

8. Define normaliser $N(H)$ of a subgroup H of a group G and show that $N(H)$ is a subgroup of G. (5)

9. Let H be a finite subgroup of a group G and let $g \in G$. Establish that the order of the subgroup gHg^{-1} = the order of H. (5)

10. Let H be a subgroup of a group G and $a \in G$. Justify that the subset $aHa^{-1} = \{aha^{-1} : h \in H\}$ is a subgroup of G. (5)

11. Explained that the polynomial $x^2 - 7$ is irreducible in $\mathbb{Q}(\sqrt{3})[x]$. (5)

12. Write down all proper subfields of $\mathbb{Q}(\sqrt{2}, \sqrt{3})$. (5)

OR

If $(\mathbb{R}, +, .)$ be a ring such that $(\mathbb{R}, +)$ is a cycle group, justify that the ring is commutative. (5)

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