14500

BRAINWARE UNIVERSITY

Term End Examination 2024-2025
Programme – M.Sc.(MATH)-2024
Course Name – Complex Analysis
Course Code - MSCMC202
(Semester II)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Compute the complex integral of cosz / z^3 over the region |z|=2.

a) πi b) -πi
c) 2πi d) -2πi

(ii) Identify the correct option-Let $f(z) = \{\frac{|z|}{z}, z \neq 0: 0, z = 0\}$ then $f(z)$

a) Has a non-zero limit as $z \to 0$
b) Is differentiable at z=0
c) Is continuous but not differentiable at z=0
d) Is neither continuous nor differentiable at z=0

(iii) Select the correct option-The cross ratio (z_1, z_2, z_3, z_4) is real iff

a) The four points lie on a circle
b) The four points lie on a straight line
c) The four points lie on a circle or on a straight line according as none of z_1, z_2, z_3, z_4 is ∞ or one of z_1, z_2, z_3, z_4 is ∞
d) None of these

(iv) Under the transformation $w = \frac{1}{z}$, the image of the line $y = \frac{1}{4}$ in the $z - plane$ visualizes

a) Circle $u^2 + v^2 + 4v = 0$
b) Circle $u^2 + v^2 = 4$
c) Straight line
d) None of these

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(v) Evaluate the residue of $\frac{1}{z(1-z^2)}$ at $z = 0$.

a) 1
b) ½
c) -1
d) 2

(vi) Select the correct option. If $f(z)$ is analytic in a simply connected domain D and C is any closed contour in D, then $\int_C f(z)dz =$

a) 0
b) 1
c) 2
d) -1

(vii) If $f(z) = \frac{e^z}{z-1}$, $z = 1$, then choose the correct option.

a) f has a simple pole at z=1 and residue at z=0 is -1
b) f has simple pole at z=1 and there exists a complex number c such that $f(z) - \frac{c}{z-1}$ is the derivative of an analytic function in a neighbour-hood of 1.
c) f has simple pole at z=1 where its residue is 1.
d) For no complex number c the function $f(z) - \frac{c}{z-1}$ is the derivative of an analytic function in a neighbour-hood of 1.

(viii) Choose the correct option. The function $f(z) = \frac{e^{z-2}}{z-2}$ has

a) A simple pole at z=2
b) A double pole at z=2
c) An essential singularity at z=2
d) A regular point at z=2

(ix) Evaluate the value of the integral $\int_C \frac{dz}{z^2-1}$, $C: |z| = 4$.

a) $-\pi i$
b) 0
c) $2\pi i$
d) None of these

(x) Let f be meromorphic function. Then select the correct option.

a) Zeroes of f are isolated points but poles are not so
b) Poles of f are isolated but zeroes are not so
c) Both poles and zeroes of f are isolated points
d) Neither zeroes nor poles of f are isolated

(xi) Select the correct value of the integral $\int_C \frac{3z^{99}+1}{z^2-1} dz$, where C is the ellipse $x^2 + 2y^2 = 8$ described in the positive sense.

a) 0
b) $2\pi i$
c) $4\pi i$
d) $6\pi i$

(xii) A Mobius transformation which transforms the upper half plane into the lower half is

a) $w = \bar{z}$
b) $w = \frac{z-1}{z+i}$
c) $w = \frac{1}{z}$
d) $w = \frac{z+i}{z-i}$

(xiii) Discuss the fixed points of $f(z) = \frac{2iz-1}{z+2i}$.

a) $1 \pm i$
b) $1 \pm 2i$
c) $\pm i$
d) $i \pm 1$

(xiv) Let f be an entire function such that $f(iy) = exp\,(-iy)\,, 0 \leq y \leq 1$. Then write the correct option.

a) $f(x+iy) = exp\,(x-iy)$, for every x and y
b) $f(x+iy) = exp\,(iy)$
c) $f(x+iy) = exp\,(x+iy)$ for every x and $0 \leq y \leq 1$
d) None of these

(xv) If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, then calculate the values of a and b.

a) $a = 2, b = -1$
b) $a = 1, b = 0$
c) $a = 0, b = 1$
d) $a = -1, b = 2$

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Justify that the power series $\sum_0^\infty n!\, z^n$ converges only for z = 0. (3)

3. Show that the $\lim\limits_{z \to 0} \frac{\bar{z}}{z}$ does not exist. (3)

4. If f(z) is a holomorphic function of z then show that (3)
$$\{\frac{\partial}{\partial x}|f(z)|\}^2 + \{\frac{\partial}{\partial y}|f(z)|\}^2 = |f'(z)|^2.$$

5. Determine $\int_C \frac{1}{z^2+2z+2} dz$, where the contour C is the unit circle |z|=1, in either direction. (3)

6. Write and justify Maximum Modulus Principle. (3)

OR

Write all the singularities of the function $f(z) = \frac{z-2}{z^2} \sin \frac{1}{z-1}$. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Justify that the function $f(z) = x^2+iy^3$, is everywhere continuous but is not analytic. (5)

8. Illustrate Cauchy Riemann Equations. (5)

9. Illustrate the Laurent's expansion of $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z+2| < 5$. (5)

10. Evaluate $\int \frac{2z+1}{z(2z+i)^2} dz$ on the circle $C: |z| = 1$. (5)

11. Write fundamental theorem of algebra and prove it. (5)

12. Justify that the circle $|w| = 1$ corresponds to the circle $x^2 + y^2 \pm 2y - 1 = 0$ under the transformation $w = \frac{1}{2}\left(z + \frac{1}{z}\right)$. (5)

OR

Write the bilinear transformation which maps z=i,1, -1 onto $w = 1, 0, \infty$ respectively. Also test that the unit circle $|z| = 1$in z-plane maps into the real-axis of w-plane. (5)

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