14501

BRAINWARE UNIVERSITY

Term End Examination 2024-2025
Programme – M.Sc.(MATH)-2024
Course Name – Partial Differential Equations
Course Code - MSCMC203
(Semester II)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Let $u(x,y)$ be the solution of the Cauchy problem $xu_x + u_y = 1, u(x,0) = 2ln\,x, x > 1$, then compute $u(e,1) =$

a) -1
b) 0
c) 1
d) e

(ii) Identify the canonical form of the differential equation $\frac{\partial^2 u}{\partial x^2} - x^2\frac{\partial^2 u}{\partial y^2} = 0$

a) $\xi = y^2 + \frac{1}{2}x, \eta = y^2 - \frac{1}{2}x$
b) $\xi = y + \frac{1}{2}x^2, \eta = y - \frac{1}{2}x^2$
c) $\xi = y + x^2, \eta = y - x^2$
d) $\xi = y^2 + x, \eta = y^2 - x$

(iii) Select the region in which the following differential equation is hyperbolic.

$$yu_{xx} + 2xyu_{xy} + xu_{yy} = u_x + u_y$$

a) $xy \neq 1$
b) $xy \neq 0$
c) $xy > 1$
d) $xy > 0$

(iv) Choose the correct option. The singular solution of the differential equation $(xp - y^2) = p^2 - 1$ is

a) $x^2 + y^2 = 1$
b) $y^2 - x^2 = 1$

c) $x^2 + 2y^2 = 1$ d) $x^2 - y^2 = 1$

(v) Identify the quasi-linear partial differential equation from the following

a) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

b) $\frac{\partial^2 u}{\partial x^2} + a(x,y)\frac{\partial^2 u}{\partial y^2} = 0$

c) $\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} = 0$

d) $\left(\frac{\partial^2 u}{\partial x^2}\right)^2 + \frac{\partial^2 u}{\partial y^2} = 0$

(vi) Select the correct option. The solution of the given differential equation $\frac{\partial^4 z}{\partial x^4} - \frac{\partial^4 z}{\partial y^4} = 0$, is

a) $f_1(y+x) + f_2(y-x) + f_3(y+ix) + f_4(y-ix)$

b) $f_1(y+x) + f_2(y-x)$

c) $f_1(y+ix) + f_2(y-ix)$

d) None of these

(vii) Select the correct option. In the case of solving a partial differential equation using separation of variable method, we equate the ratio to a constant that

a) can be positive or negative integer or zero

b) can be positive or negative integer or zero

c) must be a positive integer

d) must be a negative integer

(viii) Classify the One-dimensional wave equation.

a) $\frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\cdot\frac{\partial^2 u}{\partial x^2}$

b) $\frac{\partial^2 u}{\partial t^2} = \frac{1}{c^2}\cdot\frac{\partial^2 u}{\partial x^2}$

c) $\frac{\partial^2 u}{\partial t^2} = c^2\cdot\frac{\partial^2 u}{\partial x^2}$

d) $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\cdot\frac{\partial^2 u}{\partial t^2}$

(ix) Select the correct option. The complementary function of the equation $(2D - D' + 4)(D + 2D' + 1)^2 u = 0$ is

a) $u = e^{-2x}\phi(x - 2y) + e^{-x}[\psi_1(2x - y) + x\psi_2(2x - y)]$

b) $u = e^{-2x}\phi(x + 2y) + e^{x}[\psi_1(2x + y) + x\psi_2(2x - y)]$

c) $u = e^{-2x}\phi(x + 2y) + e^{-x}[\psi_1(2x - y) + x\psi_2(2x - y)]$

d) None of these

(x) Select the PDEs from the following that is linear.

a) $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^2 z}{\partial x^2}\frac{\partial z}{\partial y} + 8\frac{\partial^2 z}{\partial y^2} = \sin x$

b) $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^2 z}{\partial x^2}\frac{\partial z}{\partial y} + 8\frac{\partial z}{\partial x}\frac{\partial^2 z}{\partial y^2} = \sin x$

c)

d) None of these

$$\frac{\partial^2 z}{\partial x^2}+5\frac{\partial^2 z}{\partial y^2}=0$$

(xi) Select the correct option. The following PDE $9x^2y^2z = 6x^3y^2\left(\frac{\partial z}{\partial x}\right) + 6x^2y^3\left(\frac{\partial z}{\partial y}\right) + 4z\left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right)$ is

a) Non-linear
b) Linear
c) Quasi-linear
d) None of these

(xii) Choose the correct Lagrange's subsidiary equations for

$y^2zp+x^2q=xy^2$

a) $\frac{dx}{y^2z}=\frac{dy}{zx^2}=\frac{dz}{y^2}$
b) $\frac{dx}{x^2}=\frac{dy}{y^2}=\frac{dz}{zx}$
c) $\frac{dx}{1/x^2}=\frac{dy}{1/y^2}=\frac{dz}{1/zx}$
d) None of these

(xiii) Select the correct option. The general form of 3-dimensional Laplace equation is

a) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$
b) $\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}-\frac{\partial^2 u}{\partial z^2}=0$
c) $\frac{\partial^2 u}{\partial x^2}-\frac{\partial u}{\partial y}-\frac{\partial u}{\partial z}=0$
d) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$

(xiv) Choose the correct option. Solution of Heat equation when one end is insulated, then another end will be

a) Constant temperature
b) Variable temperature
c) Initial temperature
d) None of these.

(xv) Choose the correct option. The Fourier equation of Heat conduction is

a) Non-uniform
b) Finite
c) Uniform
d) Continuous

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Describe the statement of Dirichlet problem for a rectangle. (3)

3. Apply Monge's method to solve the PDE $(r-s)x=(t-s)y$ and obtain the required solution. (3)

4. Describe the characteristics of $x^2r + 2xys + y^2t = 0$. (3)

5. Explain Separation of Variables Method for the wave equation $u_{tt} - c^2u_{xx} = 0, 0 \leq x \leq L, t > 0$ subject to boundary conditions $u(0,t) = 0, u(L,t) = 0, t > 0$ and initial conditions $u(x,0) = f(x), u_t(x,0) = g(x)$ for the separation constant $k > 0$. (3)

6. Write the definition of one-dimensional heat equation. (3)

OR

Apply the method of separation of variables to solve the following problem: (3)

$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$ satisfying the following conditions

(i) $T = 0$ when $x = 0$ and 1 for all t

(ii) $T = \{2x, \quad 0 \leq x \leq \frac{1}{2} \; 2(1-x), \quad \frac{1}{2} \leq x \leq 1 \; when \, t = 0.$

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Recognize the solution of the following wave equation (5)

$$\frac{\partial^2 u}{\partial t^2} - 4\frac{\partial^2 u}{\partial x^2} = 0, -\infty < x < \infty, t > 0$$

Subject to the initial condition $u(x,0) = x, u_t(x,0) = \cos x$.

8. Consider the BVP $u_{xx} + u_{yy} = 0, x \in (0,\pi), y \in (0,\pi), u(x,0) = u(x,\pi) = u(0,y) = 0$. (5) Show that any solution of this BVP is of the form $\sum_{n=1}^{\infty} a_n \sinh n\, x \sin n\, y$.

9. Deduce the solution of the initial value problem described by PDE: $u_{tt} - c^2 u_{xx} = e^x$ with the given condition $u(x,0) = 5, \quad u_t(x,0) = x^2$ using any suitable solution method. (5)

10. Let $u = \psi(x,t)$ be the solution of the initial value problem $u_{tt} = u_{xx}$ for $-\infty < x < \infty$, $t > 0$ with $u(x,0) = \sin x$, $u_t(x,0) = \cos x$, then evaluate the value of $\psi(\frac{\pi}{2}, \frac{\pi}{6})$. (5)

11. Persuade the one-dimensional wave equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ to its canonical form. (5)

12. Construct the partial differential equation of the family of planes, the sum of whose x, y, z intercepts are equal to unity. (5)

OR

Solve and find out the integral surface of the linear PDE $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ containing the straight line $x + y = 0. z = 1$. (5)

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