14503

BRAINWARE UNIVERSITY

Term End Examination 2024-2025
Programme – M.Sc.(MATH)-2024
Course Name – General Topology
Course Code - MSCMC205
(Semester II)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Let $X=\{a,b,c,d\}$. Then select the collections of subsets that is a topological structure on X.

a) $\{\phi,X,\{a,b\},\{c\}\}$ b) $\{\phi,X,\{a,b\},\{b,c\}\}$

c) $\{\phi,X,\{a,b\},\{c,d\}\}$ d) $\{\phi,X,\{a,b\},\{c\},\{d\}\}$

(ii) Let $X=\{0,1\}$, $\tau_1=\{\phi,X,\{0\},\{1\}\}$ and $\tau_2=\{\phi,\{0,1\}\}$. Then select the correct one-

a) only τ_1 is a topology on X b) only τ_2 is a topology on X

c) both τ_1 and τ_2 are topologies on X d) neither τ_1 nor τ_2 is a topological structure on X

(iii) Select the correct one: In a topological space (X, τ), the subsets ϕ and X are

a) always open b) always compact

c) connected d) None of these

(iv) Let $X=\{a,b,c\}$. If $T_1=\{X,\phi\}$, $T_2=\{X,\phi,\{a,b\}\}$ are two topologies on X then identify the correct option from the followings.

a) T_1 is finer than T_2 b) T_2 is finer than T_1

c) T_1 and T_2 are not comparable d) $T_1=T_2$

(v) Select the correct option. $\overline{A\cup B}$ is equal to

a) $\subset \overline{A}\cup\overline{B}$ b) $=\overline{A}\cup\overline{B}$

c) $\supset \bar{A} \cup \bar{B}$ d) none of these

(vi) Select the correct option. Let X and Y be two topological spaces and $f: X \to Y$ be continuous, then for any subset A of X

a) $f(\bar{A}) \subset \overline{f(A)}$ b) $f(\bar{A}) \supset \overline{f(A)}$

c) $f(\bar{A}) = \overline{f(A)}$ d) None of these

(vii) Select the correct option $ext(A) = ?$

a) $\text{int}(A)$ b) $\text{int}(A^c)$

c) $A \setminus \text{int}(A)$ d) $A^c \setminus \text{int}(A)$

(viii) Choose the correct option: Let $X = \{a, b, c\}$ and $T = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. Then T is

a) the indiscrete topology on X b) the discrete topology on X

c) a non-Hausdorff topology on X d) a Hausdorff topology on X

(ix) Choose the correct option: the diameter of a subset A of a metric space (X, d) is

a) $Sup\{d(x,y) | (x,y) \in A\}$ b) $Inf\{d(x,y) | (x,y) \in A\}$

c) $Max\{d(x,y) | (x,y) \in A\}$ d) None of these

(x) Choose the correct option: the space $\mathbb{R}_l$ is

a) 1^{st} countable b) 2^{nd} countable

c) countable d) None of these

(xi) Select the correct option: If X is a Hausdorff metric space then the number of elements of the space is -

a) finite b) countably infinite

c) uncountable d) none of these

(xii) Select the correct option: A metric space is -

a) T_1 but not regular b) regular but not T_1

c) both T_1 and regular d) neither regular nor T_1

(xiii) Select the correct option: A T_4 space is

a) T_2 but not normal b) normal but not T_2

c) both T_2 and normal d) neither normal nor T_2

(xiv) Let X and Y are discrete topological spaces. Then select X and Y are not homeomorphic

a) only if they have same cardinality b) if they have same cardinality

c) if and only if they have same cardinality d) None of these

(xv) Select a non-homeomorphic space to the space [0, 1] from the followings.

a) (0,1) b) [2,3]

c) $[0,1) \cup (0.5, 2]$ d) $(-\infty, 2020) \cap [0, 2019]$

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Examine, whether the union $T_1 \cup T_2$ of two topologies on a set X is a topology on X or not. (3)

3. If $f: X \to Y$ and $g: Y \to Z$ are two continuous functions then show that $gof: X \to Z$ is also continuous. (3)

4. Let $S^1 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ be the unit circle in $\mathbb{R}^2$. Then establish that S^1 is closed. (3)

5. Analyze that $\{x \in \mathbb{Q} : -1 < x < 1\}$ is open in $\mathbb{Q}$ but not closed in $\mathbb{Q}$. (3)

6. Test that a discrete space X is second countable if and only if the set X is countable. (3)

OR

Justify that any closed subspace of a Lindeloff space is Lindeloff. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Let $f : (X, \tau) \to (Y, \sigma)$ be a map. Then show that f is continuous if and only if for each $x \in X$ and any neighbourhood W of $f(x)$ in Y, there exists a neighbourhood V of x such that $f(V) \subset W$. (5)

8. Let $f : X \to Y$ be a bijective continuous function. Then deduce that f is a homeomorphism if X is compact and Y is Hausdorff. (5)

9. Let (X, d) be a metric space, $x \in X$ and $r > 0$. Then analyze that the open ball $B(x, r)$ is open. (5)

10. For a subset A of a space X, examine that $\bar{A} = A \cup A'$. (5)

11. Justify that compactness is closed hereditary. (5)

12. Justify that if a topological space (X, τ) is second countable then it is Lindeloff. (5)

OR

Write an example of a 1^{st} countable space which is not 2^{nd} countable and justify. (5)

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