BRAINWARE UNIVERSITY

Term End Examination 2024-2025

Programme – B.Tech.(CSE)-2024/B.Tech.(CSE)-AIML-2024/B.Tech.(CSE)-DS-2024/B.Tech.(CSE)-AIR-2024/B.Tech.(CSE)-CYS-2024

Course Name – Algebra and Vector Calculus

Course Code - BBS00002

(Semester II)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Identify the correct option:
The rank of a matrix is always:

a) greater than equal to zero
b) greater than equal to 1
c) greater than equal to the number of row
d) none of these

(ii) For two matrices A and B of the same order, choose the correct statement

a) If rank(A) = rank(B), then A = B
b) If rank(A) < rank(B), then A = B
c) If rank(A) > rank(B), then A = B
d) None of these

(iii) Choose the maximum possible rank of the matrix of order 4X5.

a) 5
b) 4
c) 9
d) 8

(iv) Identify the correct option. If a = 3 i-2j+ 2k, b=2i-k, then (a x b). a is equal to

a) i +j+ k
b) i+ j
c) 0
d) 2

(v) Select the correct option: if i+j+k, 2i-4k, i+aj+3k are coplanar, then the value of a is

a) 2
b) 4/3
c) 5/3
d) None of these

(vi) Choose the correct option: Divergence and Curl of a vector field are

a) Scalar & Scalar
b) Scalar & Vector
c) Vector & Scalar
d) Vector & Vector

(vii) Identify the correct option.

The possible truth table entries in a Boolean function with three variables are

a) 4
b) 6
c) 8
d) 16

(viii) Select the correct option.

A canonical form of Boolean expression is written using:

a) Simplified expressions
b) Standardized minterms or maxterms
c) XOR operations
d) NAND gates

(ix) Choose the correct option.

An Boolean function A+AB simplifies to:

a) A
b) B
c) AB
d) 1

(x) An inner product in R^n is commonly defined using

a) Matrix multiplication
b) Dot product
c) Cross product
d) Determinant

(xi) If T is a linear mapping from V to W, then choose the correct option,

a) dim (Ker T) + dim (Im T)= dim V
b) dim (Ker T) + dim (Im T)= dim W
c) dim (Ker T) + dim (Im T)= dim (V + W)
d) None of these

(xii) In a vector space V over R, let $x \in R$ and $a \in V$. Then choose the correct option.

a) $x\,a \in V$
b) $x+a \in V$
c) $a.\,a \in V$
d) None of these

(xiii) Choose the correct option. Determinant of 1X1 matrix [a] is

a) a
b) 1
c) 0
d) -a

(xiv) Identify the correct option. If a square matrix of order n has n linearly independent eigenvectors, it establishes that

a) The matrix is singular
b) The matrix is diagonalizable
c) The matrix is defective
d) The matrix is zero matrix

(xv) Identify the correct option. A set of vectors is orthonormal if

a) All vectors have unit length
b) All vectors are orthogonal to each other
c) Both A and B hold
d) None of the these

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Using Cayley-Hamilton theorem, identify A^{-1} if $A=\begin{pmatrix}2 & -1\\ 1 & 3\end{pmatrix}$. (3)

3. Write the Truth table of NOR gate. (3)

4. If $\boldsymbol{A} = x^2z\hat{\imath} - 2y^3z^2\hat{\jmath} + xy^2z\hat{k}$, then calculate the value of $div\ \boldsymbol{A}$ at the point $(1,-1,1)$. (3)

5. Determine if the vectors $v_1 = (1, 2, 3)$, $v_2 = (1, 0, 1)$, $v_3 = (1, -1, 5)$ in $\mathbb{R}^3$ is linearly dependent or linearly independent. (3)

6. Test the matrix $A=\begin{pmatrix}-9 & 13\\ -2 & 6\end{pmatrix}$ is diagonalizable or not. (3)

OR

If x= - 4 is a root of $\begin{vmatrix}x & 2 & 3\\ 1 & x & 1\\ 3 & 2 & x\end{vmatrix} = 0$, evaluate the other roots. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Examine if the given matrix $A=\begin{pmatrix}-9 & 13\\ -2 & 6\end{pmatrix}$ is diagonalizable. If so, find the matrix P such that $P^{-1}AP$ is a diagonal matrix D i.e. $D = P^{-1}AP$. (5)

8. Show that (x+ y)'=x'y'. (5)

9. If $\boldsymbol{F} = grad\ (x^3 + y^3 + z^3 - 3xyz)$, then calculate the value of $div\ \boldsymbol{F}$ and $curl\ \boldsymbol{F}$. (5)

10. Identify that $\begin{vmatrix}1 & a & bc\\ 1 & b & ca\\ 1 & c & ab\end{vmatrix} = \begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}$. (5)

11. (5)

The linear transformation T: $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ is defined by T(x, y) = (x + y, x − y). Then compute Ker T^{-1}.

12. Evaluate the eigenvalues and any one eigenvector of the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$. (5)

OR

Evaluate the eigenvalues and any one eigenvector of the matrix $\begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 1 & 1 & -1 \end{bmatrix}$. (5)

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