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BRAINWARE UNIVERSITY

Term End Examination 2024-2025
Programme – M.Tech.(CSE)-AIML-2024
Course Name – Mathematics-II
Course Code - MBS00002
(Semester II)

Full Marks : 60 Time : 2:30 Hours

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Select the correct option-If f is a bounded entire function then f is

a) Non-constant
b) Not differentiable
c) Not analytic
d) Constant

(ii) In complex form identify the Cauchy-Riemann equation.

a) $\frac{\partial f}{\partial x} = i\frac{\partial f}{\partial y}$
b) $\frac{\partial f}{\partial x} = \frac{1}{i}\frac{\partial f}{\partial y}$
c) $i\frac{\partial f}{\partial x} = -\frac{\partial f}{\partial y}$
d) $\frac{\partial f}{\partial x} = -\frac{1}{i}\frac{\partial f}{\partial y}$

(iii) Identify the point/points for which $f(z) = \bar{z}$ satisfies C-R equation.

a) No z
b) All z
c) z=0
d) z=1

(iv) Tell the integral $\int_{|z|=2} \frac{\cos z}{z^3} dz$,

a) πi
b) $-\pi i$
c) $2\pi i$
d) $-2\pi i$

(v) Select the correct option-If a function is analytic at all points of a bounded domain except finitely many points, then these points are called

a) Simple points
b) singular points
c) Continuous points
d) none of these

(vi) Identify the analytic function $w = u + iv$ where $u = e^{-x}\{(x^2 - y^2)\cos y + 2xy \sin y\}$.

a) $e^{-x}\{(x - iy)^2(\cos y - i \sin y\}$
b) $e^{-x}\{(x - iy)^2(\cos y + i \sin y\}$
c) $e^{-x}\{(x + iy)^2(\cos y - i \sin y\}$
d) $e^{-x}\{(x - iy)^2(\cos y + i \sin y\}$

(vii) Identify the analytic function f(z) whose real part is $e^x \cos y$.

a) $e^z + ic$
b) e^{2z}

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c) xe^{z}

d) None of these

(viii) Select the value of the integral $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-4)(z-2)} dz$, where C is the circle $|z| = 3$and integration is taken anti-clockwise.

a) $-2\pi i$

b) πi

c) $-\pi i$

d) $2\pi i$

(ix) Identify which of the following functions is not analytic.

a) $f(z) = z^6$

b) $f(z) = \frac{1}{z^4}, z \neq 0$

c) $f(z) = \log r + i\theta$

d) $f(z) = \frac{1}{(z-1)^5}$

(x) If $f(x,y) = 0$, then solve $\frac{dy}{dx} =$

a) $\frac{f_x}{f_y}$

b) $\frac{f_y}{f_x}$

c) $-\frac{f_x}{f_y}$

d) $-\frac{f_y}{f_x}$

(xi) If $f(x,y) = x + y$ then predict $df =$

a) $dx + dy$

b) $dx - dy$

c) $dx * dy$

d) dx

(xii) Select the correct option: The convergence of gradient descent depends on:

a) The choice of the initial point

b) The step size (learning rate)

c) The properties of the function (convexity, smoothness)

d) All of the these

(xiii) Select the correct option: Which of the following problems is NOT typically solved using dynamic programming

a) Fibonacci sequence calculation

b) Shortest path in a graph (Dijkstra's algorithm)

c) 0/1 Knapsack problem

d) finding the greatest common divisor (GCD)

(xiv) Select the correct option: If the Jacobian matrix of a function is singular at a point, then the point is called a:

a) Regular point

b) Critical point

c) Ordinary point

d) Asymptotic point

(xv) Select the correct option: A singular point is classified as a saddle point if:

a) The Hessian determinant is positive

b) The Hessian determinant is negative

c) The first derivatives vanish but the function is not defined

d) The second derivatives vanish

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Show that $f(z) = xy + iy$ is nowhere analytic. (3)

3. Describe Cauchy-Riemann equation with an example. (3)

4. Write the extrema of the following function: (3)

$$f(x,y) = x^3 + 3xy^2 - 3y^2 - 3x^2 + 4$$

5. Illustrate the Euler's equation for the extremals of the functional (3)
$\int_{x_1}^{x_2}\{y^2 - yy' + y'^2\}dx$.

6. A linear mapping T: $\mathbb{R}^3 \to \mathbb{R}^4$ is defined by T (x, y, z) = (y +z , z +x, x +y, x +y +z). Evaluate Ker T. (3)

OR

If λ is an Eigen value of an orthogonal matrix, then test that $\frac{1}{\lambda}$ is also an Eigen value. (3)

Group-C

(Long Answer Type Questions)

5 x 6=30

7. Using the Gram-Schmidt orthogonalization procedure, evaluate an orthonormal basis for R^3 for the set of linearly independent vectors (2,2,0), (3,0,2), (2, -2 ,2). (5)

8. Using Residue theorem identify the value of $\oint_C \frac{z+1}{z^2-2z}dz$, where C is the circle $|z| = 5$. (5)

9. Use the substitution $r = \sqrt{x^2+y^2}$ to test that: (5)

$$\lim_{(x,y)\to(0,0)} \frac{\sin\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} = 1.$$

10. Explain the advantages and disadvantages of Newton-Raphson method (5)

11. Evaluate the eigen values and eigen vectors of the matrix A= $\begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$. (5)

12. Let v_1= (1, -1,0), v_2= (0,1, -1), v_3= (0,2,1) and v_4= (1,0,3) be elements of R^3. Justify that the set of vectors $\{v_1, v_2, v_3, v_4\}$ is linearly dependent. (5)

OR

Justify whether the set of vectors {(2,2,0), (3,0,2), (2, -2,2)} forms a basis in R^3. (5)

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