BRAINWARE UNIVERSITY

Term End Examination 2024-2025
Programme – M.Sc.(MATH)-2024
Course Name – Ordinary Differential Equations
Course Code - MSCMC104
(Semester I)

Full Marks : 60   **Time : 2:30 Hours**
[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question)   1 x 15=15

1. *Choose the correct alternative from the following :*

(i) From the followings select the non-linear differential equation.

a) $yy'' + xy' + y = x^2$

b) $y'' - y = 0$

c) $3y' + xy = e^{-x^2}$

d) None of these.

(ii) Identify the order of the differential equation $\frac{d^3y}{dx^3} - 5x\frac{dy}{dx} = e^x + 1$.

a) Third order

b) Second order

c) First order

d) None of these

(iii) Identify the value of Lipschitz constant of

$f(x,y) = x^3 \sin y, D: |x| \le 2, -\infty < y < \infty$.

a) 8

b) 24

c) 9

d) 40

(iv) Singular point of the differential equation: $(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 9y = \sin x$ is (are)_____.
Write the correct option.

a) 1

b) $x = -1$

c) $x = \pm 1$

d) None of these

(v) Write the correct option. A homogeneous differential equation is_____.

a) $2xy' + x^2y' - (\sin x)y = 2$

b) $3y' + xy = e^{-x^2}$

c) $y'' - y = 0$

d) None of these

(vi)

Select the solution of the initial value problem $y'' + 4y = 0$ given $y(0) = 0$ and $y'(0) = 1$

a) $y = \sin 2x$

b) $y = x$

c) $y = \frac{1}{2}\sin 2x$

d) None of these

(vii) Select the Wronskian of the set $\{e^x, e^{-x}\}$.

a) -2

b) -3

c) -4

d) -6

(viii) A differential equation becomes ill-behaved and one or more solutions become non-analytic at a point that is called _____. Identify the correct option.

a) Ordinary point

b) Singular point

c) Critical point

d) Special point

(ix) Select the correct one-The indicial equation is used to find:

a) The values of the singular point.

b) The values of the coefficients in the power series solution.

c) The radius of convergence of the power series solution.

d) The order of the differential equation.

(x) Choose the correct option-The radius of convergence of a power series solution is the distance from the expansion point to the:

a) Nearest singularity

b) Farthest singularity

c) Point of interest

d) Infinite series

(xi) Choose the condition necessary for a power series solution to exist for a given differential equation.

a) The differential equation must be linear

b) The differential equation must be homogeneous

c) The coefficients of the differential equation must be constant

d) The differential equation must have an analytic solution

(xii) Choose the following methods commonly used to analyze the behavior of solutions at singular points.

a) Frobenius method

b) Variation of parameters

c) Laplace transform

d) Separation of variables

(xiii) If $y = \sum_{m=0}^{\infty} a_m x^m$ is a solution of $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 3y = 0$, then write the value of $\frac{a_m}{a_{m-2}}$.

a) $\frac{(m+1)(m+2)}{m+3}$

b) $-\frac{(m+1)(m+2)}{m+3}$

c) $-\frac{m(m-1)}{m+3}$

d) $\frac{m(m-1)}{m+3}$

(xiv) If $J_0(x)$ and $J_1(x)$ are Bessel functions, then write the value of $J_1'(x)$.

a) $J_0(x) - \left(\frac{1}{x}\right)J_1(x)$

b) $-J_0(x)$

c) $J_0(x) + \left(\frac{1}{x}\right)J_1(x)$

d) $J_0(x) - \left(\frac{1}{x^2}\right)J_1(x)$

(xv) Write the value of integral $\int x^2 J_1(x)\, dx$.

a) $x^2 J_1(x) + c$

b) $x^2 J_2(x) + c$

c) $x^2 J_{-1}(x) + c$

d) $x^2 J_{-2}(x) + c$

Group-B
(Short Answer Type Questions)

3 x 5=15

2. Explain the existence and uniqueness of solutions to linear systems. (3)

3. Explain whether the boundary value problem $\frac{d^2 y}{dx^2} + \lambda(1+x)y = 0; y'(0) = 0,$ (3)

$y(1) + y'(1) = 0$ is a Sturm-Liouville problem.

4. Show that $\int_0^\infty e^{-st} L_n(t)\, dt = \left(\frac{1}{s}\right)\left(1 - \frac{1}{s}\right)^n.$ (3)

5. Write the regular singular points and irregular singular point of the following equation: (3)
$x(x-1)^2(x+2)\frac{d^2 y}{dx^2} - x^2 \frac{dy}{dx} + (x^3 + 1)y = 0.$

6. Justify the exactness of differential equation $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$ (3)

OR

Test that x=0 is an ordinary point of $(x^2 - 1)y'' + xy' - y = 0$, but x=1 is a regular singular (3)
point.

Group-C
(Long Answer Type Questions)

5 x 6=30

7. Show that $P'_{n+1} + P'_{n-1} = P_0 + 3P_1 + \cdots + (2n + 1)P_n$ (5)

8. Determine general and singular solutions of the differential equation (5)
$16x^2 + 2p^2 y - p^3 x = 0, \ p = \frac{dy}{dx}$

9. Evaluate and write all the eigen values and eigen functions of the Strum-Liouville problem (5)
$y'' + \lambda y = 0$ with $y(0) + y'(0) = 0, y(1) + y'(1) = 0$

10. Solve the ODE $(D^2 - 2D + 1)y = xe^x \sin x, D \equiv \frac{d}{dx}$ (5)

11. Apply the method of variation of parameters to solve the solution of $\frac{d^2y}{dx^2} + 4y = e^x$ (5)

12. Express the solution of $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + x^2y = 0$ in series about x=0. (5)

OR

Justify that $x^n J_n(x)$ is a solution of $x\left(\frac{d^2y}{dx^2}\right) + (1 - 2n)\left(\frac{dy}{dx}\right) + xy = 0.$ (5)

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