BRAINWARE UNIVERSITY

Term End Examination 2024-2025
Programme – B.Tech.(BT)-2024
Course Name – Calculus and Linear Algebra
Course Code - BBS00007
(Semester I)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) If f(x) satisfies all the conditions of Rolle's theorem in [a,b], then identify where f '(x) becomes zero.

a) only at one point in (a, b)
b) at two points in (a,b)
c) at least one point in (a,b)
d) none of these

(ii) Select the value of $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$

a) $\frac{2\pi}{\sqrt{3}}$
b) $\frac{3\pi}{\sqrt{2}}$
c) $\frac{\pi}{\sqrt{3}}$
d) $\frac{\pi}{\sqrt{2}}$

(iii) For $k > 0, n > 0$, Evaluate $\int_0^\infty e^{-kt} t^{n-1} dt =$

a) $\frac{\Gamma(n)}{k^n}$
b) $\frac{\Gamma(k)}{k^n}$
c) $\frac{\Gamma(k)}{n^n}$
d) $\frac{\Gamma(k)}{k}$

(iv) Choose the correct expression of $\beta(m+1, n)$ for $m > 0, n > 0$

a) $\frac{m}{m+1}\beta(m,n)$
b) $\frac{m}{m+n}\beta(m,n)$
c) $\frac{n}{m+1}B(m,n)$
d) $\frac{n}{n+1}B(m,n)$

(v) Choose the correct expression of $\beta(m,n)$ for $m > 0, n > 0$

a) $2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$
b) $2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2m-1}\theta \, d\theta$

c) $2\int_0^{\frac{\pi}{4}} sin^{2n-1}\theta\, cos^{2m-1}\theta\, d\theta$

d) None of these

(vi) Calculate $\Gamma\left(\frac{1}{2}\right) =$

a) π

b) 1

c) $\sqrt{\pi}$

d) None of these

(vii) The series $\sum_{n=1}^{\infty} \frac{1}{n^{(p+1)}}$ is divergent then select the correct value of p

a) $p \leq 0$

b) $p > 1$

c) $p > 0$

d) $p \leq 1$

(viii) Choose the correct value of $\lim_{(x,y)\to(0,0)} \frac{x^3 - y^3}{x - y} =$

a) 0

b) 1

c) 1/2

d) None of these

(ix) If $u = log\frac{x^2}{y}$ then calculate $xu_x + yu_y$=

a) u

b) 0

c) 1

d) 2u

(x) Select the rank of the zero matrix

a) 0

b) 1

c) Depends on the size of the matrix

d) Cannot be determined

(xi) If the rank of a square matrix is equal to the number of columns, then identify the type of the matrix

a) Non-invertible

b) Non-singular

c) A row matrix

d) A column matrix

(xii) If a non-diagonalizable matrix has one eigenvalue with a multiplicity of 2, establish that

a) The matrix is singular

b) The matrix is defective

c) The matrix is non-square

d) None of these

(xiii) Choose the correct option, for a positive definite matrix, the eigenvalues are:

a) All negative

b) All positive

c) All zero

d) A mix of positive and negative

(xiv) Choose the correct determinant value of a 1x1 matrix [a] from following given options

a) a

b) 1

c) 0

d) -a

(xv) Choose matrix whose all its eigenvalues equal to zero is called:

a) Nilpotent matrix

b) Diagonal matrix

c) Identity matrix

d) Invertible matrix

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Define the Maclaurin Series expansion of e^x. (3)

3. If $f(x) = sin(x)$ on the interval $[0, \pi]$, evaluate the value of c that satisfies the Mean value theorem. (3)

4. Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{2^n-1}{3^n}$. (3)

5. Determine whether the set of vectors $\{(a, b) \in \mathbb{R}^2 : 12b = 7a + 5\}$ is a vector space. (3)

6. If a+b+c $\neq 0$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ then illustrate that a = b = c. (3)

OR

If x= - 4 is a root of $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$, calculate the other roots. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Show that every square matrix can be represented as sum of symmetric and skew symmetric matrix. (5)

8. Evaluate the inverse of the matrix $\begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix}$ (5)

9. For $f(x) = \ln(x)$ on the interval $[1, e]$ calculate the value of c satisfies the Mean value theorem. (5)

10. Define that $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^4 x \, dx = \frac{3\pi}{256}$. (5)

11. Let $f(x,y) = \begin{cases} \frac{(x^2-y^2)xy}{x^2+y^2}, & (x,y) \neq 0 \\ 0, (x,y) = 0 \end{cases}$ from the definition calculate $f_{xy}(0,0)$. (5)

12. Evaluate the eigenvalues and eigenvectors of matrix A=$\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$. (5)

OR

Evaluate the kernel of the mapping $T: \mathbb{R}^3 \to \mathbb{R}^2$ is defined by $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 - x_3)$. (5)