BRAINWARE UNIVERSITY
Term End Examination 2020 - 21
Programme – Bachelor of Technology in Electronics & Communication Engineering

Course Name – Calculus

Course Code - BMAT010101

Semester / Year - Semester I

Time allotted : 85 Minutes | Full Marks : 70

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 70=70

1. *(Answer any Seventy)*

(i)

If $f(x)$ is continuous on $[2,2+h]$ and derivable in $(2,2+h)$, then $f(2+h)=f(2)+hf'(2+\theta h)$, where

a) θ is any real number

b) $-1<\theta<1$

c) $\theta>0$

d) $0<\theta<1$

(ii)

The value of $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)$ is

a)

b) $\frac{3\pi}{\sqrt{2}}$

$\frac{2\pi}{\sqrt{3}}$

c) $\frac{\pi}{\sqrt{3}}$

d) $\frac{\pi}{\sqrt{2}}$

(iii)

Which of the following pair of functions do not satisfy the Cauchy's Mean Value Theorem in the interval in [-2,2] ?

a) $x^2,\ logx$

b) $sinx^2,\ x$

c) $x-4,\ x^2+4$

d) $x^2+1,\ \frac{x}{x^2+4}$

(iv)

$$\int_{-\pi}^{\pi} \sin 6x dx =$$

a) 0

b) 1

c) -1

d) none of these

(v)

$$\int_0^\infty e^{-x^2}\,dx =$$

a) π

b) $\sqrt{\pi}$

c) $\frac{\sqrt{\pi}}{2}$

d) $\frac{\pi}{2}$

(vi)

For $k > 0,\ n > 0\ \int_0^\infty e^{-kt} t^{n-1}\,dt =$

a) $\frac{\Gamma(n)}{k^n}$

b) $\frac{\Gamma(k)}{k^n}$

c) $\frac{\Gamma(k)}{n^n}$

d) None of these

(vii)

For $m > 0,\ n > 0\ B(m+1, n) =$

a)

b)

$\frac{m}{m+1}B(m,n)$

$\frac{m}{m+n}B(m,n)$

c)

d)

$\frac{n}{m+1}B(m,n)$

None of these

(viii)

For $m>0,\ n>0\ B(m,n)=$

a)

b)

$2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta\cos^{2n-1}\theta d\theta$

$2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta\cos^{2m-1}\theta d\theta$

c)

d)

$2\int_0^{\frac{\pi}{2}} \sin^{2n-1}\theta\cos^{2m-1}\theta d\theta$

None of these

(ix)

$\Gamma\left(\frac{1}{2}\right)=$

a)

b) 1

π

c)

d)

None of these

$\sqrt{\pi}$

(x)

$\lim_{x \to 0+} x \log x =$

a) 1

b) 0

c) 2

d)

None of these

(xi)

The Cauchy's form of remainder in Taylor's theorem is

a)

$$\frac{h^n(1-\theta)^{(n-1)}}{(n-1)!} f^n(a+\theta h)$$

b)

$$\frac{h^n(1-\theta)^{(n-p)}}{p(n-1)!} f^n(a+\theta h)$$

c)

$$\frac{h^n}{n!} f^n(a+\theta h)$$

d)

None of these

(xii)

The Schlomilch-Roche's form of remainder in Maclaurin's theorem is

a)

$$\frac{x^n(1-\theta)^{(n-1)}}{(n-1)!} f^n(\theta x)$$

b)

$$\frac{x^n(1-\theta)^{(n-p)}}{p(n-1)!} f^n(\theta x)$$

c)

d)

$$\frac{x^n}{n!} f^n(\theta x)$$

None of these

(xiii)

The Lagrange's form of remainder in Maclaurin's theorem is

a)

$$\frac{x^n(1-\theta)^{(n-1)}}{(n-1)!} f^n(\theta x)$$

b)

$$\frac{x^n(1-\theta)^{(n-p)}}{p(n-1)!} f^n(\theta x)$$

c)

$$\frac{x^n}{n!} f^n(\theta x)$$

d)

None of these

(xiv)

$$\int_0^1 \frac{1}{\sqrt{1-x^2}} dx =$$

a)

π

b) 1

c)

$$\frac{\pi}{2}$$

d)

None of these

(xv)

The improper integral $\int_0^1 \frac{\sin\frac{1}{x}}{\sqrt{x}}\,dx$ is

a) Convergent

b) Divergent

c) 1

d) None of these

(xvi)

The improper integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx =$

a) π

b) 1

c) $\frac{\pi}{2}$

d) None of these

(xvii)

The improper integral $\int_1^{\infty} \frac{1}{x\left(1+x^2\right)}\,dx$ is

a) Convergent

b) Divergent

c) 1

d) None of these

(xviii)

The improper integral $\int_1^\infty f(x)dx$, where $f(x)=\begin{cases} \frac{1}{x^2}, \text{ if } x \text{ be rational} \geq 1 \\ -\frac{1}{x^2}, \text{ if } x \text{ be irrational} > 1 \end{cases}$ is

a) Convergent

b) Divergent

c) 1

d) None of these

(xix)

The characteristic points of the circles $(x-\alpha)^2+y^2=\alpha^2$ are

a) $(\alpha, \pm a)$

b) $(\pm\alpha, a)$

c) $(\pm\alpha, -a)$

d) None of these

(xx)

The sequence $\left\{\frac{1}{3^n}\right\}$

a) Monotonic increasing

b)

Oscillatory

c) Monotonic decreasing

d) None of these

(xxi)

If $\sum_{n=1}^{\infty} x_n$ is convergent, then the series $\sum_{n=1}^{k} u_n + \sum_{n=1}^{\infty} x_n$ is

a) convergent

b) divergent

c) oscillatory

d) nothing can be said

(xxii)

Second term of sequence with general term n^2 - 4/2 is

a) 2

b) -3

c) 1

d) 0

(xxiii)

Sum of an infinite geometric series exist only if condition on common ratio r is

a) $-1 < r < 1$

b) $-1 \leq r \leq 1$

c)

d)

$r < -1, r > 1$ $r \le -1, r \ge 1$

(xxiv)

The Sequence $\left\{1, \frac{1}{5}, \frac{1}{5^2}, \ldots, \frac{1}{5^n}, \ldots \infty\right\}$ is

a) divergent

b) oscillatory

c) convergent

d) none of these

(xxv)

The sequence $\{4, 4, 4, \ldots\}$ is called a

a) Monotone increasing sequence

b) Monotone decreasing sequence

c) Constant sequence

d) None of these

(xxvi)

The sequence $\{x_n\}$, where $x_n = (-1)^{n-1}$, is

a) a convergent sequence

b) a divergent sequence

c) an oscillatory sequence

d) None of these

(xxvii)

The sequence $\{x_n\}$, where $x_n = \frac{1}{2}\{1+(-1)^{n-1}\}$, is a sequence with

a)

n terms

b)

2 terms

c)

No terms

d)

None of these

(xxviii)

The sequence $\left\{1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots\right\}$ is

a)

Bounded

b)

Unbounded

c)

Divergent

d)

None of these

(xxix)

The sequence $\left\{\frac{1}{n}\right\}$ is

a)

Convergent sequence

b)

Divergent sequence

c)

Oscillating sequence

d)

None of these

(xxx)

$$\lim_{n\to\infty}\left(2+\left(-\frac{1}{2}\right)^n\right)=$$

a) 2

b) 0

c) 1

d) None of these

(xxxi)

The sequence $\left\{\frac{1}{n^p}\right\}$, where $p>0$ is

a) Null sequence

b) Divergent sequence

c) Constant sequence

d) None of these

(xxxii)

For $|x|>1$ and $p>0$, $\lim_{n\to\infty}\frac{x^n}{n!}=$

a) 0

b) 1

c) 2

d) None of these

(xxxiii)

The sequence $\left\{\frac{1}{n}\right\}$ is

a)

b)

Bounded sequence

Unbounded sequence

c) Divergent sequence

d) None of these

(xxxiv)

The sequence $\left\{(-1)^n\right\}$ is

a) Bounded sequence

b) Unbounded sequence

c) Convergent sequence

d) None of these

(xxxv)

A convergent sequence is

a) Bounded sequence

b) Unbounded sequence

c) Oscillating sequence

d) None of these

(xxxvi)

The sequence $\left\{\frac{n^2+1}{n^2}\right\}$ converges to

a) 0

b) 1

c) 1/2

d) None of these

(xxxvii)

$\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)=$

a) 1

b) 0

c) 3

d) None of these

(xxxviii)

The sequence $\left\{-n^2\right\}$ is

a) Convergent

b) Bounded

c) Divergent

d) None of these

(xxxix)

$\lim a^{\frac{1}{n}}=1$ if

a) $0<a<1$

b) $|r|\leq 1$

c) $a>0$

d) None of these

(xl)

The sequence $\left\{\frac{n}{n+1}\right\}$ is

a)

An unbounded sequence

b)

Anoscillating sequence

c)

A convergent sequence

d)

None of these

(xli)

The series $1+a+a^2+\ldots$ is convergent only when

a)

$|a|<1$

b)

$|a|\leq 1$

c)

$|a|\geq 1$

d)

None of these

(xlii)

The series $\sum \frac{1}{n^p}$ is convergent for

a) p >1

b)

$p\leq 1$

c) p = 0

d)

None of these

(xliii)

The series $\frac{1+2}{2^3}+\frac{1+2+3}{3^3}+\frac{1+2+3+4}{4^3}+\ldots$ is

a) Convergent

b) Divergent

c) Absolutely convergent

d) None of these

(xliv)

The series $\sum u_n$, where $u_n=\sqrt{n^4+1}-\sqrt{n^4-1}$ is

a) Convergent

b) Divergent

c) Absolutely divergent

d) None of these

(xlv)

The series $x+\frac{x^2}{2}+\frac{x^3}{3}+\ldots$, $x>0$ is divergent if

a) $x<1$

b) $x\geq 1$

c) $-1<x<0$

d) None of these

(xlvi)

The series $1+\frac{1}{1!}+\frac{2^2}{2!}+\frac{3^3}{3!}+\ldots$ is

a) Convergent

b) Divergent

c) Absolutely convergent

d) None of these

(xlvii)

The series $1-\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}-\frac{1}{5^2}-\frac{1}{6^2}+\ldots$ is

a) Convergent

b) Divergent

c) Absolutely divergent

d) None of these

(xlviii)

For n>0, $\int_{-\pi}^{\pi} \cos nx\,dx =$

a) 1

b) 0

c) 3π

d) 4π

(xlix)

Sum of Fourier series of the function $f(x)=x+x^2, -\pi<x<\pi$ at the point $x=\pi$ is:

a)

$\pi+\pi^2$

b)

π

c)

π^2

d) 0

(l)

Take the "odd function" out:

a)

$f(x)=x$

b)

$f(x)=|x|$

c)

$f(x)=x\sin x$

d)

$f(x)=e^{-|x|}$

(li)

If $u=\log(x^2+y^2)$, then $u_{xx}+u_{yy}=$

a) 0

b)

$\frac{x}{y}$

c)

d) 1

$\frac{y}{x}$

(lii)

For an odd function, the Fourier series expansion contains

a) only cosine terms

b) only sine terms

c) bosine and cosine terms

d)

a. none of these

(liii)

If $f(x,y)=0$, then $\frac{dy}{dx}=$

a) $\frac{f_x}{f_y}$

b) $\frac{f_y}{f_x}$

c) $-\frac{f_x}{f_y}$

d) $-\frac{f_y}{f_x}$

(liv)

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{x^2-y^2}{x^2+y^2} =$$

a) 0

b) 1

c) 1/2

d)

none of these

(lv)

If $u(x,y)=yf(\frac{x^2}{y^2})$ then show that $x\frac{\delta u}{\delta x}+y\frac{\delta u}{\delta x}=$

a) 0

b)

$2u(x,y)$

c)

d) 2

$u(x,y)$

(lvi)

$$\int_1^3\int_2^{-1}\int_1^2 dxdydz =$$

a) -6

b) 3

c) -3

d) 2

(lvii)

$$\int_2^3\int_4^6 dxdy =$$

a) 1
b) 12
c) 2
d) 0

(lviii)

$$\int_0^1\int_y^{\sqrt{y}} dxdy =$$

a) 1/2
b) 1/6
c) 2/3
d) 4/3

(lix)

The value of $\iiint xyzdxdydz$ over $R:[0,1;0,1;0,1]$ is

a) $-\frac{2}{3}a^4$

b) $-\frac{4}{3}a^4$

c) $\frac{1}{8}$

d) $-\frac{2}{3}a^5$

(lx)

The value of $\iint_R x^3 ydxdy$ over the region $R:\{0\le x\le 1;\ 0\le y\le 2\}$ is

a) 1/2

b) 8/945

c) 16/45

d) 16/945

(lxi)

The value of $\int_C (x\,dx - dy)$,where C is the line joining (0,1) to (1,0) is

a) 3/2

b) 1/2

c) 0

d) 2/3

(lxii)

If $f = 2x^2 - 3y^2 + 4z^2$, then curl(grad f) =

a) $4x\hat{i} - 6y\hat{j} + 8z\hat{k}$

b) $x\hat{i} + y\hat{j} + z\hat{k}$

c) $\vec{0}$

d) 3

(lxiii)

If $\vec{a}.(\vec{b} \times \vec{c}) = 0$, then the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are

a) independent

b) coplanar

c) collinear

d) none of these

(lxiv)

If θ be the angle between the vectors $\vec{a}=6\hat{i}+2\hat{j}+3\hat{k}$ & $\vec{b}=2\hat{i}-9\hat{j}+5\hat{k}$ then

a)

$\theta=\cos^{-1}\left(\frac{9}{7\sqrt{110}}\right)$

b)

$\theta=\tan^{-1}\left(\frac{12}{77}\right)$

c) both (a) and (b)

d) none of these

(lxv)

If $u=x^4+4y^3z$, then $grad(u)=$

a)

$4x^3\hat{i}+12y^2z\hat{j}+4y^3\hat{k}$

b)

$4x^3\hat{i}+12y^2z\hat{j}+4y^2\hat{k}$

c)

$4x^3\hat{i}+12yz\hat{j}+4y^2\hat{k}$

d)

$4x\hat{i}+12yz\hat{j}+4y^2\hat{k}$

(lxvi)

$\vec{A}=2xz^2\hat{i}-yz\hat{j}+3xz^3\hat{k}$ and $\phi=x^2yz$, then $curl\left(curl\ \vec{A}\right)=$

a)

$\left(-4x-9z^2\right)\hat{i}+\left(4z+1\right)\hat{k}$

b)

$\left(-4x+9z^2\right)\hat{i}+\left(4z-1\right)\hat{k}$

c)

d)

$\left(-4x-9z^2\right)\hat{i}-\left(4z+1\right)\hat{k}$

$\left(-4x+9z^2\right)\hat{i}-\left(4z+1\right)\hat{k}$

(lxvii)

If $\phi(x,y,z)=xy+yz+zx$, then $\vec{\nabla}\phi=$

a)

$(y+z)\hat{i}+(z+x)\hat{j}+(x+y)\hat{k}$

b)

$(y+z)\hat{i}-(z+x)\hat{j}+(x+y)\hat{k}$

c)

$(y+z)\hat{i}-(z+x)\hat{j}-(x+y)\hat{k}$

d)

$(y+z)\hat{i}+(z+x)\hat{j}-(x+y)\hat{k}$

(lxviii)

$\nabla.\left(\vec{E}\times\vec{F}\right)=$

a)

$\vec{F}.\nabla\times\vec{E}-\vec{E}.\nabla\times\vec{F}$

b)

$\vec{F}.\nabla\times\vec{F}-\vec{E}.\nabla\times\vec{F}$

c)

$\vec{F}.\nabla\times\vec{F}-\vec{E}.\nabla\times\vec{E}$

d)

None of these

(lxix)

The improper integral $\int_0^\infty e^{-px}dx$, p is a constant, converges for

a) <div> p>0</div>

b) p<0

c) p=0

d) none of these

(lxx)

The sequence $\left\{ \frac{1}{n^2} \right\}$ is

a) bounded sequence

b) unbounded sequence

c) divergent sequence

d) none of these