BRAINWARE UNIVERSITY
Term End Examination 2020 - 21
Programme – Bachelor of Technology in Electronics & Communication Engineering
Course Name – Calculus
Course Code - BSC(ECE)101
Semester / Year - Semester I

Time allotted : 75 Minutes Full Marks : 60

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 60=60

1. *(Answer any Sixty)*

(i)

If f(x) satisfy all the conditions of Rolle's theorem in $[a,b]$, then $f'(x)$ becomes zero

a) only at one point in (a, b)

b) at two points in (a,b)

c) at least one point in (a,b)

d) none of these

(ii)

If $f(x)$ is continuous on $[2,2+h]$ and derivable in $(2,2+h)$, then $f(2+h)=f(2)+hf'(2+\theta h)$, where

a) θ is any real number

b) $-1<\theta<1$

c)

d)

$\theta > 0$

$0 < \theta < 1$

(iii)

$$\int_{-\pi}^{\pi} \sin 6x dx =$$

a) 0

b) 1

c) -1

d)

none of these

(iv)

$$\int_{0}^{\infty} e^{-x^2} dx =$$

a)

π

b)

$\sqrt{\pi}$

c)

$\frac{\sqrt{\pi}}{2}$

d)

$\frac{\pi}{2}$

(v)

For $k > 0, n > 0 \int_{0}^{\infty} e^{-kt} t^{n-1} dt =$

a)

b)

$\frac{\Gamma(n)}{k^n}$

$\frac{\Gamma(k)}{k^n}$

c)

d) None of these

$\frac{\Gamma(k)}{n^n}$

(vi)

For $m > 0,\ n > 0\ B(m+1, n) =$

a)

b)

$\frac{m}{m+1} B(m, n)$

$\frac{m}{m+n} B(m, n)$

c)

d)

$\frac{n}{m+1} B(m, n)$

None of these

(vii)

For $m > 0,\ n > 0\ B(m, n) =$

a)

b)

$2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta d\theta$

$2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2m-1}\theta d\theta$

c)

d)

None of these

$$2\int_0^{\frac{\pi}{2}} \sin^{2n-1}\theta \cos^{2m-1}\theta \, d\theta$$

(viii)

$$\Gamma\left(\frac{1}{2}\right) =$$

a) π

b) 1

c) $\sqrt{\pi}$

d) None of these

(ix)

$$\lim_{x\to 0+} x\log x =$$

a) 1

b) 0

c) 2

d) None of these

(x)

If a function $f:[a,b]\to\mathbb{R}$ be continuous on $[a,b]$ and differentiable on (a,b). If $f'(x)=0$ for all $x\in(a,b)$ then the value of f is

a) 1

b) 0

c)

d)

Constant

None of these

(xi)

The Cauchy's form of remainder in Taylor's theorem is

a)

$$\frac{h^n(1-\theta)^{(n-1)}}{(n-1)!} f^n(a+\theta h)$$

b)

$$\frac{h^n(1-\theta)^{(n-p)}}{p(n-1)!} f^n(a+\theta h)$$

c)

$$\frac{h^n}{n!} f^n(a+\theta h)$$

d)

None of these

(xii)

The Lagrange's form of remainder in Maclaurin's theorem is

a)

$$\frac{x^n(1-\theta)^{(n-1)}}{(n-1)!} f^n(\theta x)$$

b)

$$\frac{x^n(1-\theta)^{(n-p)}}{p(n-1)!} f^n(\theta x)$$

c)

$$\frac{x^n}{n!} f^n(\theta x)$$

d)

None of these

(xiii)

$$\int_0^1 \frac{1}{\sqrt{1-x^2}} dx =$$

a)

b) 1

π

c)

d)

$\frac{\pi}{2}$

None of these

(xiv)

The improper integral $\int_0^1 \frac{\sin\frac{1}{x}}{\sqrt{x}}\,dx$ is

a)

b)

Convergent

Divergent

c) 1

d)

None of these

(xv)

The improper integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx =$

a)

b) 1

π

c)

d)

$\frac{\pi}{2}$

None of these

(xvi)

The improper integral $\int_1^{\infty} \frac{1}{x(1+x^2)} dx$ is

a) Convergent

b) Divergent

c) 1

d) None of these

(xvii)

The improper integral $\int_1^{\infty} f(x)dx$, where $f(x) = \begin{cases} \frac{1}{x^2}, \text{ if } x \text{ be rational} \geq 1 \\ -\frac{1}{x^2}, \text{ if } x \text{ be irrational} > 1 \end{cases}$ is

a) Convergent

b) Divergent

c) 1

d) None of these

(xviii)

The characteristic points of the circles $(x-\alpha)^2 + y^2 = \alpha^2$ are

a) $(\alpha, \pm a)$

b) $(\pm\alpha, a)$

c)

d)

$(\pm\alpha, -a)$

None of these

(xix)

The sequence $\left\{\frac{1}{3^n}\right\}$

a)

Monotonic increasing

b)

Oscillatory

c)

Monotonic decreasing

d)

None of these

(xx)

If $\sum_{n=1}^{\infty} x_n$ is convergent, then the series $\sum_{n=1}^{k} u_n + \sum_{n=1}^{\infty} x_n$ is

a)

convergent

b)

divergent

c)

oscillatory

d)

nothing can be said

(xxi)

Second term of sequence with general term n^2 - 4/2 is

a) 2

b) -3

c) 1 d) 0

(xxii)

Sum of an infinite geometric series exist only if condition on common ratio r is

a) $-1 < r < 1$

b) $-1 \leq r \leq 1$

c) $r < -1, r > 1$

d) $r \leq -1, r \geq 1$

(xxiii)

The Sequence $\left\{1, \frac{1}{5}, \frac{1}{5^2}, \ldots\ldots, \frac{1}{5^n}, \ldots\ldots\infty\right\}$ is

a) divergent

b) oscillatory

c) convergent

d) none of these

(xxiv)

The sequence $\{4, 4, 4, \ldots\}$ is called a

a) Monotone increasing sequence

b) Monotone decreasing sequence

c) Constant sequence

d) None of these

(xxv)

The sequence $\{x_n\}$, where $x_n = (-1)^{n-1}$, is

a) a convergent sequence

b) a divergent sequence

c) an oscillatory sequence

d) None of these

(xxvi)

The sequence $\{x_n\}$, where $x_n = \frac{1}{2}\left\{1+(-1)^{n-1}\right\}$, is a sequence with

a) n terms

b) 2 terms

c) No terms

d) None of these

(xxvii)

The sequence $\left\{1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots\right\}$ is

a) Bounded

b) Unbounded

c) Divergent

d) None of these

(xxviii)

The sequence $\left\{\frac{1}{n}\right\}$ is

a)

Convergent sequence

b)

Divergent sequence

c)

Oscillating sequence

d)

None of these

(xxix)

$$\lim_{n\to\infty}\left(2+\left(-\frac{1}{2}\right)^{n}\right)=$$

a) 2

b) 0

c) 1

d)

None of these

(xxx)

The sequence $\left\{\frac{1}{n^{p}}\right\}$, where $p>0$ is

a)

Null sequence

b)

Divergent sequence

c)

Constant sequence

d)

None of these

(xxxi)

For $|x| > 1$ and $p > 0$, $\lim_{n \to \infty} \frac{x^n}{n!} =$

a) 0

b) 1

c) 2

d)

None of these

(xxxii)

The sequence $\{(-1)^n\}$ is

a)

Bounded sequence

b)

Unbounded sequence

c)

Convergent sequence

d)

None of these

(xxxiii)

A convergent sequence is

a)

Bounded sequence

b)

Unbounded sequence

c)

Oscillating sequence

d)

None of these

(xxxiv)

The sequence $\left\{\frac{n^2+1}{n^2}\right\}$ converges to

a) 0

b) 1

c) 1/2

d)

None of these

(xxxv)

The sequence $\{-n^2\}$ is

a) Convergent

b) Bounded

c) Divergent

d) None of these

(xxxvi)

$\lim a^{\frac{1}{n}} = 1$ if

a) $0 < a < 1$

b) $|r| \leq 1$

c) $a > 0$

d) None of these

(xxxvii)

The sequence $\left\{\frac{n}{n+1}\right\}$ is

a) An unbounded sequence

b) Anoscillating sequence

c)

d)

A convergent sequence

None of these

(xxxviii)

The series $1 + a + a^2 + ...$ is convergent only when

a) $|a| < 1$

b) $|a| \leq 1$

c) $|a| \geq 1$

d) None of these

(xxxix)

The series $\sum \frac{1}{n^p}$ is convergent for

a) p >1

b) $p \leq 1$

c) p = 0

d) None of these

(xl)

The series $\sum u_n$, where $u_n = \sqrt{n^4 + 1} - \sqrt{n^4 - 1}$ is

a) Convergent

b) Divergent

c)

d)

Absolutely divergent

None of these

(xli)

For n>0, $\int_{-\pi}^{\pi} \cos nx dx =$

a) 1

b) 0

c) 3π

d) 4π

(xlii)

Take the “odd function” out:

a) $f(x) = x$

b) $f(x) = |x|$

c) $f(x) = x \sin x$

d) $f(x) = e^{-|x|}$

(xliii)

If $f(x, y) = 0$, then $\frac{dy}{dx} =$

a) $\frac{f_x}{f_y}$

b) $\frac{f_y}{f_x}$

c)

d)

$-\frac{f_x}{f_y}$

$-\frac{f_y}{f_x}$

(xliv)

If u $=\log\frac{x^2}{y}$ then $xu_x + yu_y =$

a) u

b) b

c) 1

d) 2u

(xlv)

If $u(x,y) = yf(\frac{x}{y^2}^2)$ then show that $x\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta x} =$

a) 0

b)

$2u(x,y)$

c)

d) 2

$u(x,y)$

(xlvi)

$\int_2^3\int_4^6 dxdy =$

a) 1

b) 12

c) 2

d) 0

(xlvii)

$$\int_0^1 \int_y^{\sqrt{y}} dxdy =$$

a) 1/2

b) 1/6

c) 2/3

d) 4/3

(xlviii)

The value of $\iint_R xy\left(x^2+y^2\right)dxdy$ over $R:[0,a;0,b]$.

a)

$ax^2/2 - x^5/30$

b)

$ax^2/2 - x^3/6$

c)

$ax^2/2$

d)

$\frac{1}{8}a^2b^2\left(a^2+b^2\right)$

(xlix)

Find the value of $\iint xydxdy$ over the area bounded by the parabola $x = 2a$ and $x^2 = 4ay$, is

a)

$\frac{a^4}{4}$

b)

$\frac{a^4}{3}$

c)

d)

$\frac{a^3}{3}$ $\frac{a^2}{3}$

(l)

The value of $\iiint xyz\,dx\,dy\,dz$ over $R:[0,1;0,1;0,1]$ is

a) $-\frac{2}{3}a^4$

b) $-\frac{4}{3}a^4$

c) $\frac{1}{8}$

d) $-\frac{2}{3}a^5$

(li)

The value of $\iint_R x^3 y\,dx\,dy$ over the region $R:\{0 \le x \le 1;\ 0 \le y \le 2\}$ is

a) 1/2
b) 8/945
c) 16/45
d) 16/945

(lii)

The value of $\int_C (x\,dx - dy)$, where C is the line joining (0,1) to (1,0) is

a) 3/2
b) 1/2
c) 0
d) 2/3

(liii)

If $\vec{a}.(\vec{b} \times \vec{c}) = 0$, then the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are

a) independent

b) coplanar

c) collinear

d) none of these

(liv)

If θ be the angle between the vectors $\vec{a} = 6\hat{i} + 2\hat{j} + 3\hat{k}$ & $\vec{b} = 2\hat{i} - 9\hat{j} + 5\hat{k}$ then

a) $\theta = \cos^{-1}\left(\frac{9}{7\sqrt{110}}\right)$

b) $\theta = \tan^{-1}\left(\frac{12}{77}\right)$

c) both (a) and (b)

d) none of these

(lv)

If $u = x^4 + 4y^3 z$, then $grad(u) =$

a) $4x^3\hat{i} + 12y^2z\hat{j} + 4y^3\hat{k}$

b) $4x^3\hat{i} + 12y^2z\hat{j} + 4y^2\hat{k}$

c) $4x^3\hat{i} + 12yz\hat{j} + 4y^2\hat{k}$

d) $4x\hat{i} + 12yz\hat{j} + 4y^2\hat{k}$

(lvi)

$\vec{A} = 2xz^2\hat{i} - yz\hat{j} + 3xz^3\hat{k}$ and $\phi = x^2yz$, then $curl\left(curl\ \vec{A}\right) =$

a)

$\left(-4x - 9z^2\right)\hat{i} + \left(4z + 1\right)\hat{k}$

b)

$\left(-4x + 9z^2\right)\hat{i} + \left(4z - 1\right)\hat{k}$

c)

$\left(-4x - 9z^2\right)\hat{i} - \left(4z + 1\right)\hat{k}$

d)

$\left(-4x + 9z^2\right)\hat{i} - \left(4z + 1\right)\hat{k}$

(lvii)

If $\phi(x, y, z) = xy + yz + zx$, then $\vec{\nabla}\phi =$

a)

$(y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$

b)

$(y+z)\hat{i} - (z+x)\hat{j} + (x+y)\hat{k}$

c)

$(y+z)\hat{i} - (z+x)\hat{j} - (x+y)\hat{k}$

d)

$(y+z)\hat{i} + (z+x)\hat{j} - (x+y)\hat{k}$

(lviii)

$\nabla.\left(\vec{E} \times \vec{F}\right) =$

a)

$\vec{F}.\nabla \times \vec{E} - \vec{E}.\nabla \times \vec{F}$

b)

$\vec{F}.\nabla \times \vec{F} - \vec{E}.\nabla \times \vec{F}$

c) $\vec{F}.\nabla\times\vec{F}-\vec{E}.\nabla\times\vec{E}$

d) None of these

(lix)

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx =$$

a) π

b) $\sqrt{\pi}$

c) $\frac{\sqrt{\pi}}{2}$

d) $\frac{\pi}{2}$

(lx)

The sequence $\left\{\frac{1}{n^2}\right\}$ is

a) bounded sequence

b) unbounded sequence

c) divergent sequence

d) none of these