# BRAINWARE UNIVERSITY
## Term End Examination 2020 - 21
**Programme – Master of Science in Mathematics**
**Course Name – Functional Analysis**
**Course Code - MSCMC301**
**Semester / Year - Semester III**

Time allotted : 75 Minutes    Full Marks : 60

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

## Group-A

(Multiple Choice Type Question)    1 x 60=60

1. *(Answer any Sixty )*

(i) Which of the following statement is true?

a) Every metric space is a normed space
b) Every normed space is a metric space
c) Every vector space is metric space
d) Every metric space is a vector space

(ii) If a normed space is a Banach Space, then which of the following is false

a) Every sequence is convergent
b) Every Cauchy sequence is convergent
c) Every convergent sequence is Cauchy
d) Every closed set is complete

(iii) A normed space is a Banach Space if and only if

a) Every sequence is convergent
b) Every Cauchy sequence is convergent
c) Every convergent sequence is Cauchy
d) None of these

(iv) The dimension of the set of all real numbers with usual addition and multiplication is

a) Is not a vector space
b) 1
c) Finite dimension but not 1
d) Infinite dimensional

(v) Let Y be a closed subspace of a Banach space X. Then Y is

a) Compact
b) Complete
c) convex but not complete
d) None of these

(vi) If a Banach space X has a basis. Then X is

a) Separable metric space

b) Inseparable metric space

c) Both Separable metric space and Inseparable metric space can be possible

d) Either Separable metric space or Inseparable metric space always true

(vii) Let Y be a finite-dimensional subspace of a normed space X then Y

a) Closed

b) Open

c) Both open and closed

d) Neither open nor close

(viii) The value of a norm on a complex vector space is

a) a vector

b) a pure real number

c) a complex number

d) other

(ix) If the norm of a vector is zero then what can we say about the vector

a) the vector is orthogonal to zero vector

b) vector is the zero vector

c) vector can be a non-zero vector but parallel to zero vector

d) other

(x) Equivalent norm on a normed space define

a) same topology

b) comparable but not same topology

c) incomparable topology

d) Does not always define a topology

(xi) Let X be a finite dimensional normed space. Then two normed on X are

a) equivalent

b) incomparable

c) equivalent if two normed are complete

d) None of these

(xii) In a finite dimensional normed space X. A subset M is compact if

a) M is only closed

b) M is only bounded

c) M is both closed and bounded

d) M may not be closed or bounded

(xiii) In a metric space X. A subset M is compact then

a) M is only closed
b) M is only bounded
c) M is both closed and bounded
d) M may not be closed or bounded

(xiv) In a metric space X. A subset M is compact if

a) M is only closed
b) M is only bounded
c) M is both closed and bounded
d) None of these

(xv) A mapping between two metric spaces from X to Y is called an open map if

a) Image of every open set of X is open in Y
b) Image of every subsets of X is open in Y
c) Pre-image of every open set of Y is open in X
d) Pre-image of every subsets of Y is open in X

(xvi) Let X and Y are Banach spaces and T from D to Y a closed linear operator, where D is a closed subset of X. Then which of the following is false?

a) T is bounded
b) T is continuous
c) T takes a Cauchy sequence to a Cauchy sequence
d) None of these

(xvii) The null space of a closed linear operator is

a) Closed
b) Open
c) Clopen
d) None of these

(xviii) The inverse of a closed linear operator is always

a) Closed
b) closed if the operator is bounded
c) closed if domain is closed
d) closed if domain is complete

(xix) The domain of a linear functional is always a

a) Metric space
b) Vector space
c) Topological space
d) Any non-empty subset of C

(xx) Let X be a finite dimensional vector space and dim(X)=5. Then dim(X**) is

a) <5  
b) >5  
c) 5  
d) other

(xxi) Let X be a normed linear space then dual space X' is a

a) Banach space  
b) Banach space only if X is Banach  
c) Banach space if X is complete  
d) None of these

(xxii) Let p be a positive homogeneous functional then

a) p(0)=1  
b) p(0)=2  
c) p(0) is either 0 or 2  
d) p(0) never equals to 1

(xxiii) The addition of two sublinear functional is a

a) subadditive functional but not a sublinear functional  
b) positive homogeneous functional but not a sublinear functional  
c) sublinear functional  
d) linear functional

(xxiv) Let p and q are two sublinear functional then 2p+3q is a

a) subadditive functional but not a sublinear functional  
b) positive homogeneous functional but not a sublinear functional  
c) sublinear functional  
d) linear functional

(xxv) Every sublinear functional induced a

a) Pseudonorm  
b) seminorm  
c) norm  
d) None of these

(xxvi) Let Y be a closed subspace of a Hilbert space X. Then Y is

a) Compact  
b) Complete  
c) convex but not complete  
d) convex but not compact

(xxvii) If a Hilbert space X has a basis. Then X is

a) Separable metric space

b) Inseparable metric space

c) Both Separable metric space and Inseparable metric space can be possible

d) Either Separable metric space or Inseparable metric space always true

(xxviii) Let Y be a finite-dimensional subspace of a Hilbert space X then Y

a) Closed

b) Open

c) Both open and closed

d) Neither open nor close

(xxix) In an inner product space of dimension 2 if a set of two vectors are orthogonal to each other then the set:

a) Linearly dependent

b) Linearly independent but not a basis

c) Basis

d) None of these

(xxx) Which is false? Given that two Hilbert spaces are isomorphic to each other

a) only if they have same Hilbert dimension

b) if they have same Hilbert dimension

c) if and only if they have same Hilbert dimension

d) Other

(xxxi)

Let $\|.\|$ be a norm on a vector space X. Then $\|x+y\|$

a)

$=\|x\|+\|y\|$

b)

$\geq\|x\|+\|y\|$

c)

$\leq\|x\|+\|y\|$

d)

None of these

(xxxii)

The value of the norm of the continuous function $f:[0,1] \to R, f(x) = 2x$ , where $\|f\| = \max_x |f(x)|$

a) 0
b) 1
c) 2
d) 4

(xxxiii)

Let d be a metric induced by a norm on a normed space X. Then which is always true?

a) $d(-2x,-2y) \leq 2d(x,y)$
b) $d(-2x,-2y) \leq -2d(x,y)$
c) $d(-2x,-2y) = 2d(x,y)$
d) $d(-2x,-2y) + 2d(x,y) = 0$

(xxxiv)

If $\|(x,y)\| = (|x|^3 + |y|^3)^{\frac{1}{3}}$ be a normed defined over the vector space $R^2$. Then $\|(-1,1)\| = ?$

a) 0
b) $2^{\frac{1}{3}}$
c) 2
d) 8

(xxxv)

Which of the following is not a subspace of $R^2$ ?

a) $\{(x,0): x \in R\}$
b) $\{(0,y): y \in R\}$

c)

$\{(x,1): x \in R\}$

d)

$\{(x,y): x = y; x,y \in R\}$

(xxxvi)

Let $\{x_n\}$ be a sequence convergent to $x$ in a normed space X. Then which is true?

a)

$\|x_n\| = \|x\|, \forall n$

b)

$\|x_n\| = \|x\|$ for finitely many n.

c)

$\|x_n\| = \|x\|$ for infinitely many n.

d)

$\|x_n - x\| \to 0, n \to \infty$

(xxxvii)

Let $T : X \to Y$ be a bounded linear operator then which is always true?

a)

$\|Tx\| = \|x\|$

b)

$\|Tx\| \leq \|x\|$

c)

$\|Tx\| \geq c\|x\|$

d)

$\|Tx\| \leq c\|x\|$

(xxxviii)

Let $T : X \to Y$ be a continuous linear operator between normed spaces $X$ and $Y$. Then the null space of T is

a) Open

b) Closed

c)

d) None of these

both closed and open

(xxxix)

A bounded, linear and one-one operator T from a Banach space X onto a Banach space Y be such that for every sequence $\{x_n\} \to 0$ in X implies $\{Tx_n\} \to 0$ in Y. Then

a) T is continuous in X

b) T is only continuous at 0 in X

c) T is continuous in X except at 0.

d) None of these

(xl)

Let X and Y are two NLS and $T: D(T) \to Y$ be called a closed linear operator. If $x_n \in X : x_n \to x \Rightarrow T(x_n) \to y$ then

a) $T(x) = y$

b) $T(x_n) = y, \forall n$

c) $T(x_n) = y$ all but finitely many n

d) $T(x_n) = T(x), \forall n$

(xli)

Let X and Y are two NLS and $T: D(T) \to Y$ be called a closed linear operator. If $\{x_n\}$ and $\{x'_n\}$ are two sequences in X converges to the same limit then

a) $T(x_n) = T(x'_n), \forall n$

b) $T(x_n) = T(x'_n)$ for infinitely many $n$

c)

d)

$\{T(x_n)\} \wedge \{T(x'_n)\} \to y$

No relation between $T(x_n), T(x'_n)$.

(xlii)

Let X and Y are two NLS and $T: D(T) \to Y$ be called a closed linear operator. Then $D(T)$ is closed if

a)

T is bounded

b)

Y is complete

c)

Both T is bounded and Y is complete

d)

None of these

(xliii)

Let X be a complete metric space and $U_n, n \geq 1$ be a collection of open dense subsets of X. Then the intersection $\bigcap_{n \geq 1} U_n$ in X

a)

dense

b)

nowhere dense

c)

empty set

d) Other

(xliv)

A subset M in a metric space is called nowhere dense in X if

a)

$\overline{M}$ has no accumulation points

b)

$\overline{M}$ has no interior points

c)

d)

$\overline{M} \neq X$ Other

(xlv)

To apply uniform boundedness theorem over the sequence of bounded linear operator $\{T_n\} \in B(X,Y)$. Then we consider X as a

a) only an NLS

b) only a complete space

c) Banach space

d) Other

(xlvi)

Let $f$ be a bounded linear functional on an NLS X then which of the following is always true?

a) $\|f(x)\| \leq \|x\|$

b) $\|f(x)\| \leq \|f\|\|x\|$

c) $\|f(x)\| \geq \|x\|$

d) $\|f(x)\| \geq \|f\|\|x\|$

(xlvii)

Find the value of $\|f\|$, where $f: R^3 \to R$ defined by $f(x,y,z) = x+y+z, \forall (x,y,z) \in R^3$

a) 1

b) 3

c) $\sqrt{3}$

d) other

(xlviii)

Let X be a vector space and dimension of X is n. Then the dimension of the dimension of the algebraic dual X* of X is

a) $\leq n$

b) $\geq n$

c) $n$

d) Other

(xlix)

Which of the following condition is true for sublinear functional?

a) $p(x+y) = p(x) + p(y)$

b) $p(x+y) \leq p(x) + p(y)$

c) $p(x+y) \geq p(x) + p(y)$

d) other

(l)

Let p be a positive homogeneous function then

a) $p(2x) = 2p(x)$

b) $p(2x) \leq 2p(x)$

c) $p(2x) \geq 2p(x)$

d) Other

(li)

Which of the following is a linear functional?

a) $p(x,y) = (x,0)$

b) $p(x,y) = x$

c) $p(x,y) = x + y - 1$

d) $p(x,y,z) = (x,y,z)$

(lii)

Let X be a real vector space and p a sub linear functional on X. Furthermore, let f be a linear functional which is defined on a subspace Z of X and satisfies $f(x) \leq p(x)$ for all $x$ belongs to Z. Then

a) there exist an extension of f to a linear functional on X.

b) there exist an extension of f to a sub linear functional on X.

c) there exist an extension of f to a subadditive functional on X.

d) there exist an extension of f to a positive homogeneous functional on X.

(liii)

For every $x$ in a normed linear space X,.

a) $\|x\| \leq \sup_{f \in X', f \neq 0} \frac{|f(x)|}{\|f\|}$

b) $\|x\| = \sup_{f \in X', f \neq 0} \frac{|f(x)|}{\|f\|}$

c)

d) None of these

$$\|x\| \geq \sup_{f \in X', f \neq 0} \frac{|f(x)|}{\|f\|}$$

(liv)

Which of the following is true for any sublinear functional?

a)

$|p(x) - p(y)| \leq p(x - y)$

b)

$|p(x) - p(y)| \geq p(x - y)$

c)

$|p(x) - p(y)| = p(x - y)$

d)

None of these

(lv)

Which of the following set is an orthonormal set?

a)

$\{(1,0),(0,2)\}$

b)

$\{(1,0),(-1,1)\}$

c)

$\left\{\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right),\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)\right\}$

d)

$\left\{\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right),\left(\frac{1}{\sqrt{2}},0\right)\right\}$

(lvi)

For a complex inner product space, we have

a)

$\operatorname{Re}(x.y) = \frac{1}{4}\left(\|x+y\|^2 - \|x-y\|^2\right)$

b)

$\operatorname{Re}(x.y) = \frac{1}{4}\left(\|x+y\| + \|x-y\|\right)$

c)

$$\mathrm{Re}(x.y)=\frac{1}{4}(\|x+y\|-\|x-y\|)^2$$

d)

$$\mathrm{Re}(x.y)=\frac{1}{4}(\|x\|^2-\|y\|^2)$$

(lvii)

A linear operator on a finite dimensional complex normed space with at least two elements has

a)

at least one eigen value

b)

exactly one eigen value

c)

exactly two eigen values

d)

at most one eigen value

(lviii)

Let E be a subset of a separable inner product space. Then E is

a)

separable

b)

separable if E is a subspace

c)

separable if E is a closed subspace

d)

other

(lix)

The dual space of $l^1$ is

a) $l^{\infty}$

b) $l^{1}$

c) $l^{1/2}$

d) Other

(lx)

Let X be a finite dimensional vector space and dim(X)=1. Then dim(X*) is

a) >1

b) <1

c) 1

d) Not assigned