BRAINWARE UNIVERSITY
Term End Examination 2020 - 21
Programme – Master of Science in Mathematics
Course Name – Differential Geometry
Course Code - MSCMC302
Semester / Year - Semester III

Time allotted : 75 Minutes Full Marks : 60

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 60=60

1. *(Answer any Sixty)*

(i) The intrinsic derivative of the fundamental tensor is

a) Constant b) Undefined

c) 0 d) None of these

(ii) If a curve lies on a plane, then it is called a

a) Skew curve b) Plane curve

c) Straight line d) None of these

(iii) A geodesic curve

a) Is an auto parallel curve b) Is not an auto parallel curve

c) A parallel curve d) None of these

(iv) In Euclidean space, geodesic are

a) Circle b) Parabola

c) Straight lines d) None of these

(v) If in a Riemannian space, there exist a coordinate system with respect to which the Christoffel symbols vanish at a given point then that system is called

a) Rectangular system b) Spherical system

c) Geodesic coordinate system d) None of these

(vi) If the geodesic curvature of a curve on a surface is zero then the curve is called

a) Gaussian curve b) Geodesic curve

c) A curve d) None of these

(vii) The right helicoid is a surface of

a) Positive curvature b) Negative curvature

c) Zero curvature d) None of these

(viii) The total curvature of a surface depends only on the

a) First fundamental form b) Second fundamental form

c) Third fundamental form d) None of these

(ix) A straight line on a surface is

a) Asymptotic lines b) Tangent line

c) Not a asymptotic line d) None of these

(x) For a minimal surface, the asymptotic lines at a point are

a) Parallel b) Mutually orthogonal

c) In a constant direction d) None of these

(xi) A tensor whose all components are zero, is called a

a) Null tensor b) Zero tensor

c) Constant tensor d) None of these

(xii) By the process of outer multiplication of two tensors followed by a contraction, we get a new tensor. This tensor is called

a) Outer product b) Inner product

c) Product d) None of these

(xiii) If by interchanging two contravariant indices of a tensor, each of its components is altered in sign but not in magnitude, then the tensor is said to be

a) Symmetric
b) Skew-symmetric
c) Constant
d) None of these

(xiv) In geodesic coordinate system the point at which the Christoffel symbols vanishes is called

a) Origin
b) Pole
c) Null point
d) None of these

(xv)

The intrinsic derivative of a vector A along a curve C vanishes at all points of C, then the magnitude of A along the curve C is

a) 0
b) Constant
c) Not specific
d) None of these

(xvi)

Serret – Frenet formulae for a space curve are

a)

$$\mu^i = \frac{1}{\kappa}\frac{\delta\lambda^i}{\delta s} \; and$$

$$\nu^i = \frac{1}{\tau}\left(\frac{\delta\mu^i}{\delta s} + \kappa\lambda^i\right)$$

b)

$$\mu^i = \frac{1}{\kappa}\frac{\delta\lambda^i}{\delta s} \; and \; \frac{\delta\nu^k}{\delta s} = -\tau\mu^k$$

c)

d) None of these

$$\mu^i = \frac{1}{\kappa}\frac{\delta\lambda^i}{\delta s},\ \nu^i = \frac{1}{\tau}\left(\frac{\delta\mu^i}{\delta s} + \kappa\lambda^i\right)$$

$$and\ \frac{\delta\nu^k}{\delta s} = -\tau\mu^k$$

(xvii)

The plane determined by λ and μ is called the osculating plane at $P(x_0^i)$ and its equation is

a)

$$g_{ij}\left(x^i - x_0^i\right)\nu^j = 0$$

b)

$$\mu^i = \frac{1}{\kappa}\frac{\delta\lambda^i}{\delta s}$$

c)

$$\nu^i = \frac{1}{\tau}\left(\frac{\delta\mu^i}{\delta s} + \kappa\lambda^i\right)$$

d)

$$\frac{\delta\nu^k}{\delta s} = -\tau\mu^k$$

(xviii)

The first fundamental quadratic form of the surface is

a)

$$a_{\alpha\beta}du^\alpha du^\beta,$$

$$where\ a_{\alpha\beta} = \sum_i \frac{\partial x^i}{\partial u^\alpha}\frac{\partial x^i}{\partial u^\beta},$$

$$\alpha, \beta = 1, 2$$

b)

$$\frac{1}{\tau}\left(\frac{\delta\mu^i}{\delta s} + \kappa\lambda^i\right)$$

c)

$$g_{ij}\left(x^i - x_0^i\right)\nu^j$$

d)

$$-\tau\mu^k$$

(xix)

ds^2 for the surface $x^1 = u^1, x^2 = u^2, x^3 = f(u^1, u^2)$ is

a)

$$ds^2 = (1+f_1^2)(du^1)^2 + 2f_1 f_2 du^1 du^2 + (1+f_2^2)(du^2)^2$$

b)

$$ds^2 = (2+f_1^2)(du^1)^2 + 2f_2 du^1 du^2 + (1+f_2^2)(du^2)^2$$

c)

$$ds^2 = (2+f_1^2)(du^1)^2 + 2f_2 du^1 du^2$$

d)

None of these

(xx)

The length ds of the element of arc joining two neighbouring points on the surface with position vectors r and $r+dr$ is

a)

$$ds^2 = r^2{}_u (du)^2 + 2r_u r_v du dv + r_v^2 (dv)^2$$

b)

$$ds^2 = (du)^2 + 2du dv + (dv)^2$$

c)

$$ds^2 = (du)^2 + (dv)^2$$

d)

None of these

(xxi)

For the paraboloid $r = (u, v, u^2 - v^2)$

a)

$$ds^2 = (du)^2 + (u^2 + 1)(dv)^2$$

b)

$$ds^2 = (du)^2 + (u^2 + v^2)(dv)^2$$

c)

$$ds^2 = (1 + f_1^2)(du)^2 + u^2(dv)^2$$

d)

$$ds^2 = (1 + 4u^2)(du)^2 - 4uvdudv + (1 + 4v^2)(dv)^2$$

(xxii)

If θ be the angle between two parametric curves, then $\cos\theta =$

a)

$$\frac{a_{12}}{\sqrt{a_{11}a_{22}}}$$

b)

$$\frac{a_{11}}{\sqrt{a_{11}a_{22}}}$$

c)

$$\frac{a_{22}}{\sqrt{a_{11}a_{22}}}$$

d)

None of these

(xxiii)

The parametric curves on a surface are orthogonal if and only if

a)

$a_{12} = 0$

b)

$a_{11} = 0$

c)

d) None of these

$a_{22} = 0$

(xxiv)

The differential equation of the geodesic in rectangular coordinates is

a) $\frac{d^2 x^i}{ds^2} = 0$

b) $\frac{d}{dt}\left(\frac{\partial \varphi}{\partial \dot{u}^\alpha}\right) = 0$

c) $\frac{\partial \varphi}{\partial u^\alpha} = 0$

d) None of these

(xxv)

The Gaussian curvature of a surface S is

a) $K = \frac{R_{1212}}{a}$, $a = |a_{\alpha\beta}|$.

b) $K = \frac{R_{1112}}{a}$, $a = |a_{\alpha\beta}|$.

c) $K = \frac{R_{1111}}{a}$, $a = |a_{\alpha\beta}|$.

d) $K = \frac{R_{2121}}{a}$, $a = |a_{\alpha\beta}|$.

(xxvi)

The Gaussian curvature K for a surface is

a)

b)

An invariant

Not an invariant

c)

d)

Always zero

None of these

(xxvii)

The properties of the curvature K of a surface are

a)

b)

Intrinsic properties

Not intrinsic properties

c)

d)

Geodesic properties

None of these

(xxviii)

Two surfaces S_1 and S_2 be such that there exists a coordinate system with respect to which the linear element on S_1 and S_2 are characterised by the same metric tensor, then they are said to be

a)

b) Isometric

Non isometric

c)

d)

No specific terminology

None of these

(xxix)

Let $C: u^{\alpha} = u^{\alpha}(s)$ be a curve on the surface S whose metric tensor is given by $ds^2 = a_{\alpha\beta} du^{\alpha} du^{\beta}$. In the relation $\frac{\delta \lambda^{\beta}}{\delta s} = \kappa_g \eta^{\beta}$, the scalar κ_g is called

a) The geodesic curvature of C

b) The Gaussian curvature of C

c) The curvature of C

d) None of these

(xxx)

The quadratic form $\mathbb{C} = c_{\alpha\beta} du^{\alpha} du^{\beta}$, where $c_{\alpha\beta} = g_{ij} \xi^{i}{}_{,\alpha} \xi^{j}{}_{,\beta}$ is the

a) Second fundamental form of the surface

b) Third fundamental form of the surface

c) First fundamental form of the surface

d) None of these

(xxxi)

$H = \frac{1}{2} a^{\alpha\beta} b_{\alpha\beta}$ is called the

a) Geodesic curvature of the surface

b) Mean curvature of the surface

c) Curvature of the surface

d) None of these

(xxxii)

If $H=0$ then the surface is called a

a) Minimal surface

b) Surface

c) Gaussian surface

d) None of these

(xxxiii)

The normal curvatures in the direction of the coordinate curves are respectively

a) $\frac{b_{11}}{a_{11}}$ and $\frac{b_{22}}{a_{22}}$

b) $\frac{b_{12}}{a_{12}}$ and $\frac{b_{22}}{a_{22}}$

c) $\frac{b_{12}}{a_{12}}$ and $\frac{b_{22}}{a_{22}}$

d) $\frac{b_{12}}{a_{12}}$ and $\frac{b_{21}}{a_{21}}$

(xxxiv)

On a surface S the directions for which $\kappa_{(n)}$ has the extreme values are called the

a) Normal directions at the given point

b) Principal directions at the given point

c) Tangential directions at the given point

d) None of these

(xxxv)

A point on a surface is parabolic if

a)

$\kappa_{(1)}$ and $\kappa_{(2)}$ have opposite signs.

b)

$\kappa_{(1)}$ and $\kappa_{(2)}$ are zero.

c)

$\kappa_{(1)}$ and $\kappa_{(2)}$ are of same signs.

d) None of these

(xxxvi)

Rodrigue's formula is

a)

$$\kappa_{(\rho)} \frac{dx^{r}}{ds} = 0$$

b)

$$\frac{\delta \xi^{\varepsilon r}}{\delta s} + \kappa_{(\rho)} \frac{dx^{r}}{ds} = 0$$

c)

$$\frac{\delta \xi^{\varepsilon r}}{\delta s} = 0$$

d)

None of these

(xxxvii)

The lines of curvature on a surface are given by

a)

$$\in^{\gamma\delta} a_{\alpha\gamma} b_{\beta\delta} du^{\alpha} du^{\beta} = 0$$

b)

$$\in^{\gamma\delta} a_{\alpha\beta} b_{\gamma\delta} du^{\alpha} du^{\beta} = 0$$

c)

d) None of these

$$\in^{\alpha\beta} a_{\alpha\beta} b_{\gamma\delta} du^{\alpha} du^{\beta} = 0$$

(xxxviii)

If K is the Gaussian curvature of a surface, then Enneper's formula is

a)

$Torsion\ of\ an$
$asymptotic\ line = \pm\sqrt{-K}$

b)

$Torsion\ of\ an$
$asymptotic\ line = \pm\sqrt{K}$

c)

$Torsion\ of\ an$
$asymptotic\ line = \sqrt{K}$

d)

None of these

(xxxix)

The equation for the principal curvatures of the surface for the right helicoids is

a)

$$\kappa_{(\rho)} = \pm\frac{c}{u^2 + c^2}$$

b)

$$\kappa_{(\rho)} = \frac{c}{u^2 + c^2}$$

c)

$$\kappa_{(\rho)} = \frac{c}{u^2}$$

d)

None of these

(xl)

For a curve C lying on a surface S, if A_i is a space vector defined along C, whose parameter is t, then $\frac{\delta A_i}{\delta t} =$

a)

$$\frac{dA_i}{dt} - {}_g\left\{ {i \atop j \quad k} \right\} A_j \frac{dx^k}{dt}$$

b)

$$\frac{dA_i}{dt} - {}_g\left\{ {i \atop j \quad i} \right\} A_j \frac{dx^k}{dt}$$

c)

$$\frac{dA_i}{dt} - {}_g\left\{ {i \atop k \quad i} \right\} A_j \frac{dx^k}{dt}$$

d)

$$\frac{dA_i}{dt} - {}_g\left\{ {j \atop i \quad k} \right\} A_j \frac{dx^k}{dt}$$

(xli)

A surface is plane, if

a)

$b_{\alpha\beta} = 0$

b)

$b_{\alpha\beta} = 1$

c)

$b_{\alpha\beta} = \text{constant}$

d)

None of these

(xlii)

The rectangular Cartesian coordinates of the point whose cylindrical coordinates are $\left(2, \frac{\pi}{3}, 1\right)$ are

a)

$\left(1, \sqrt{3}, 1\right)$

b)

$(1, 2, 1)$

c) $(1,3,1)$

d) None of these

(xliii)

The rectangular Cartesian coordinates of the point whose cylindrical coordinates are $\left(4,\frac{\pi}{3},1\right)$ are

a) $\left(\frac{6}{\sqrt{2}},\frac{6}{\sqrt{2}},1\right)$

b) $\left(4\sqrt{3},4,2\right)$

c) $\left(2,2\sqrt{3},1\right)$

d) None of these

(xliv)

$\delta^i{}_k\delta^k{}_l\delta^l{}_i =$

a) 0

b) 1

c) n

d) None of these

(xlv)

$\delta^i{}_j\delta^k{}_l A^{jl} =$

a) 1

b)

c) A^{ik}

d) None of these

A^{i}

(xlvi)

If $x^{i}{}_{,i} B_i = 0$, B_i any arbitrary covariant vector, then $x^{i}{}_{,i} =$

a) n

b) 0

c) 1

d) None of these

(xlvii)

Let ψ be a function of n corodinates $\left(x^1, x^2, \ldots, x^n\right)$ and $\bar{\psi}$ be its transformation on change of coordinates $\left(x^1, x^2, \ldots, x^n\right)$ to $\left(\bar{x}^1, \bar{x}^2, \ldots, \bar{x}^n\right)$, then ψ is called invariant with respect to the transformation if $\frac{\partial \bar{\psi}}{\partial \bar{x}^i} =$

a) $\frac{\partial \psi}{\partial x^j} \cdot \frac{\partial x^j}{\partial \bar{x}^i}$

b) $\frac{\partial \psi}{\partial x^i} \cdot \frac{\partial x^i}{\partial \bar{x}^j}$

c) $\frac{\partial x^i}{\partial \bar{x}^j}$

d) None of these

(xlviii)

The contracted tensor $A^m{}_n$ is a

a) Scalar

b) Vector

c) 1

d) None of these

(xlix)

The kronecker delta $\delta^i{}_j$ is

a) Symmetric in i and j

b) Skew-symmetric in i and j

c) 1

d) None of these

(l)

If A_m is a covariant vector , then $\frac{\partial A_m}{\partial x^s} - \frac{\partial A_s}{\partial x^m}$ is

a) Symmetric

b) Skew-Symmetric

c) Constant

d) None of these

(li)

The maximum number of independent components of the Christofell symbols in an m-dimensional Riemannian space is

a)

b)

$^{m}C_2$ $\frac{m^2(m+1)}{2}$

c) m

d) None of these

(lii)

A surface S on which the Gaussian curvature vanishes at every point is called a

a) Flat surface

b) Gaussian surface

c) Developable surface

d) None of these

(liii)

On a surface if the curvature $K=0$, then the surface is said to be

a) Flat surface

b) Developable

c) Euclidean

d) None of these

(liv)

The parametric curves are the lines of curvature if and only if

a) $a_{12}=b_{12}=0$

b) $a_{22}=b_{22}=0$

c) $a_{11}=b_{11}=0$

d) None of these

(lv)

The quadratic form $g_{mn}dx^m dx^n$ is

a) Positive definite

b) Positive semidefinite

c) Negative definite

d) None of these

(lvi)

If $g_{ij} = 0$ for $i \neq j$, then $\begin{Bmatrix} i \\ i \quad i \end{Bmatrix} =$

a) 0

b) n

c) $\frac{1}{2} \cdot \frac{\partial}{\partial x^i} \log g_{ii}$

d) None of these

(lvii)

If $g_{ij} = 0$ for $i \neq j$, then $\begin{Bmatrix} i \\ j \quad j \end{Bmatrix} =$

a) $-\frac{1}{2g_{ii}} \cdot \frac{\partial g_{jj}}{\partial x^i}$

b) 0

c) n

d) None of these

(lviii)

If $g_{ij} = 0$ for $i \neq j$, then $[i\,i, i] =$

a) 1

b) n

c) 0

d)

$$\frac{1}{2}\cdot\frac{\partial g_{ii}}{\partial x^i}$$

(lix)

If $g_{ij} = 0$ for $i \neq j$, then $[i\,j, i] =$

a)

$$\frac{1}{2}\cdot\frac{\partial g_{ii}}{\partial x^i}$$

b)

$$-\frac{1}{2g_{ii}}\cdot\frac{\partial g_{jj}}{\partial x^i}$$

c)

$$-\frac{1}{2}\cdot\frac{\partial g_{ii}}{\partial x^k}$$

d)

$$\frac{1}{2}\cdot\frac{\partial g_{ii}}{\partial x^j}$$

(lx)

Which of the following set span the vector space $\left\{\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} : x, y \in R\right\}$?

a)

$$\left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$$

b)

$$\left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\}$$

c)

$$\left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}\right\}$$

d)

$$\left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}\right\}$$