BRAINWARE UNIVERSITY
Term End Examination 2020 - 21
Programme – Master of Science in Mathematics
Course Name – Integral Equations & Calculus of Variations
Course Code - MSCMC303
Semester / Year - Semester III

Time allotted : 75 Minutes Full Marks : 60

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 60=60

1. *(Answer any Sixty)*

(i)

In $g(x)y(x) = f(x) + \lambda \int_a K(x,t)y(t)dt$, the upper limit

a) may be the any value

b) may be variable of x

c) may be fixed constant

d) Either may be variable of x or May be fixed constant

(ii)

The function K(x,t) of $g(x)y(x) = f(x) + \lambda \int_a K(x,t)y(t)dt$ is called….. of the integral equation.

a) Kernel

b) integral

c) integral constant

d) None of these

(iii)

In $g(x)y(x) = f(x) + \lambda \int_a^{\square} K(x,t)y(t)dt$, if g(x)≠ 0, then it is called

a) linear integral equation of 1^{st} kind

b) linear integral equation of 2^{nd} kind

c) linear integral equation of 3^{rd} kind

d) None of these

(iv)

If g(x)=0, in $g(x)y(x) = f(x) + \lambda \int_a^{\square} K(x,t)y(t)dt$, then the equation is called

a) linear integral equation of 1^{st} kind

b) linear integral equation of 2^{nd} kind

c) linear integral equation of 3^{rd} kind

d) None of these.

(v)

Which of the following is Fredholm Integral equation ?

a) $g(x)y(x) = f(x) + \lambda \int_a^b K(x,t)y(t)dt$

b) $g(x)y(x) = f(x) + \lambda \int_a^{\square} K(x,t)y(t)dt$

c)

d)

$y(x) = \int_a^b K(x,t)[y(t)]^2 dt$ None of these.

(vi)

Fredholm Integral equation of first kind is

a) $f(x) + \lambda \int_a^b K(x,t)y(t)dt = 0$

b) $f(x) + \lambda \int_a^{\square} K(x,t)y(t)dt = 0$

c) $y(x) = f(x) + \lambda \int_a^{\square} K(x,t)y(t)dt$

d) None of these.

(vii)

Homogeneous Fredholm Integral equation of second kind is

a) $y(x) = f(x) + \lambda \int_a^{\square} K(x,t)y(t)dt$

b) $y(x) = f(x) + \lambda \int_a^b K(x,t)y(t)dt$

c) $y(x) = \lambda \int_a^b K(x,t)y(t)dt$

d) None of these

(viii)

Volterra integral equation of 2^{nd} kind is

a) $y(x) = f(x) + \lambda \int_a^x K(x,t)y(t)dt$

b) $y(x) = f(x) + \lambda \int_a^b K(x,t)y(t)dt$

c) $y(x) = f(x) + \int_a^x K(x,t)y(t)dt$

d) None of these

(ix)

A real kernel $K(x,t)$ is………

a) skew-symmetric

b) symmetric

c) not symmetric

d) None of these

(x)

Which of the following is symmetric kernel?

a) $\sin(x+t)$

b) $\sin(2x+3t)$

c) x^2t^3+1

d) None of these

(xi)

A kernel $K(x,t)$ is called separable kernel if it can be expressed as

a) $K(x,t) = \sum_{i=0}^{n} g_i(x)h_i(t)$

b) $K(x,t) = \sum_{i=1}^{n} g_i(x)h_i(t)$

c) $K(x,t) = \sum_{i=1}^{\infty} g_i(x)h_i(t)$

d) None of these

(xii)

For what value of λ, the function $y(x) = 1 + \lambda x$ is a solution of the integral equation $x = \int_0^x e^{x-t} y(t)dt$?

a) $\lambda = 1$

b) $\lambda = 2$

c) $\lambda = -1$

d) $\lambda = -2$

(xiii)

Initial value problem is always converted into

a)

b)

Fredholm integral equation Volterra integral equation

c) d) None of these

Fredholm integral equation and Volterra integral equation

(xiv)

The eigen values of the integral equation $y(x) = \lambda \int_0^{2\pi} \sin(x+t)\, y(t)dt$ are

a) b)

$\frac{1}{2\pi}, -\frac{1}{2\pi}$ $\frac{1}{\pi}, -\frac{1}{\pi}$

c) d)

$\pi, -\pi$ $2\pi, -2\pi$

(xv)

Which of the following function is a solution of Fredholm integral equation $f(x) = x + \int_0^1 xtf(t)dt$?

a) b)

$\frac{2x}{3}$ $\frac{3x}{2}$

c) d)

$\frac{3x}{4}$ $\frac{4x}{3}$

(xvi)

The I.VP corresponding to the integral equation $y(x) = 1 + \int_0^x y(t)dt$ is a

a) $y' - y = 0 \, , y(0) = 1$

b) $y' + y = 0 \, , y(0) = 0$

c) $y' - y = 0 \, , y(0) = 0$

d) $y' + y = 0 \, , y(0) = 1$

(xvii)

When an ordinary differential equation is to be solved under conditions involving dependent variable and its derivative at different values of the independent variable, the problem

a) Boundary value problem

b) Initial value problem

c) only ODE

d) None of these.

(xviii)

The integral equation corresponding to the IVP $y'' + xy' + y = 0 \; with y(0) = 1, y'(0) = 0$

a) $u(x) = -1 - \int_0^1 (2x - t)u(t)dt$

b) $u(x) = -1 - \int_0^x (2x - t)u(t)dt$

c)

$u(x) = -1 - \int_1^x (2x - t)u(t)dt$

d)

$u(x) = 1 - \int_0^x (2x - t)u(t)dt$

(xix)

The values of λ for which the integral equation $y(x) = \lambda \int_0^1 (6\text{x} - \text{t})y(t)dt$ has a non-trivial solution , are given by the roots of the equations:

a)

$(3\lambda - 1)(2 + \lambda) - \lambda^2 = 0$

b)

$(3\lambda - 1)(2 + \lambda) + 2 = 0$

c)

$(3\lambda - 1)(2 + \lambda) - 4\lambda^2 = 0$

d)

$(3\lambda - 1)(2 + \lambda) + \lambda^3 = 0$

(xx)

The eigen values of the integral equation $y(x) = \lambda \int_0^1 (2\text{xt} - 4x^2)y(t)dt$ are

a)

3,3

b)

-3,-3

c)

3,-3

d)

-3,3

(xxi)

The value of λ for which the solution of the equation $y(x) = cosx + \lambda \int_0^\pi sinx\, y(t)dt$ is $y(x) = cosx$, is a)

$\lambda \neq 1$

b)

$\overline{\lambda \neq 2}$

c)

$\lambda \neq \frac{1}{2}$

d) None of these

(xxii)

A mapping $f: X \to X$ is a contraction mapping if there exists a constant k such that

a)

$d(x,y) = kd(f(x), f(y))$

b)

$d(x,y) \leq kd(f(x), f(y))$

c)

$d(x,y) \geq kd(f(x), f(y))$

d) other

(xxiii)

The kernel for the volterra equation $\phi(x) = x + \lambda \int_a (x-s)\phi(s)ds$ is

a)

(x+s)

b)

e^{x-s}

c)

(x-s)

d) None of these

(xxiv)

The resolvent kernel of the integral equation $u(x)=1+\int_0^x u(t)dt$ is

a) e^{x-t}

b) $e^{-(x-t)}$

c) e^{x+t}

d) None of these

(xxv)

The eigen values λ of the integral equation $y(x)=\lambda\int_0^\pi \sin(x+t)y(t)\,dt$ are

a) $\frac{1}{2\pi},-\frac{1}{2\pi}$

b) $\frac{1}{\pi},-\frac{1}{\pi}$

c) $\pi,-\pi$

d) $2\pi,-2\pi$

(xxvi)

The kernel of the integral equation $y(x)=x-\int_0^x xt^2y(t)dt, x>0$ is

a)

b)

tx^2

c)

t^2x^2

xt^2

d) None of these

(xxvii)

The kernel for the volterra equation $\phi(x) = x + \lambda \int_a^x \phi(s)\,ds$ is

a) 0

b) 2

c) 1

d) None of these

(xxviii)

The function $K(x,y)$ is said to be symmetric if

a)

$K(x,y) = K(y,x)$

b)

$K(x,y) = -K(y,x)$

c)

$K(x,y) = 0$

d) None of these

(xxix)

The expression of non-linear Fredholm –Hammerstein Integral Equation of Second
Second kind is

a)

b)

$$y(x)=\int_a^b K(x,t)F(y(t))\,dt$$

$$y(x)=f(x)+\int_a^b K(x,t)F(y(t))\,dt$$

c)

$$y(x)=\int_0^x K(x,t)F(y(t))\,dt$$

d)

$$y(x)=f(x)+\int_0^x K(x,t)F(y(t))\,dt$$

(xxx)

The solution to Abel equation is given by

a)

$$\phi(x)=\frac{\sin\alpha\pi}{\pi}$$

b)

$$\phi(x)=\frac{\sin\alpha\pi}{\pi}\left[\frac{f(0)}{x^{1-\alpha}}+\int_0^x\frac{f'(y)}{(x-y)^{1-\alpha}}dy\right]$$

c)

$$\phi(x)=x$$

d) None of these

(xxxi)

Let $K(x,y)$ be a given real function defined for $0\le x\le 1,\ 0\le y\le 1$ and $f(x)$ a realvalued function defined for $0\le x\le 1$ and λ an arbitrary complex number. Then $\phi(x)-\lambda\int^1 K(x,y)\phi(y)dy=f(x),\ (0\le x\le 1)$ is the

a)

b)

Linear Fredholm integral equation of second kind(for a function $\phi(x)$)

Linear Fredholm integral equation of first kind(for a function $\phi(x)$)

c)

Volterra integral equation of first kind(for a function $\phi(x)$)

d)

Volterra integral equation of second kind(for a function $\phi(x)$)

(xxxii)

In linear integral equation $y(x) = F(x) + \lambda \int_a^b k(x,t) y(t)\, dt$. If a and b are constants. The equation is known as

a)

Fredholm integral equation

b)

Volterra integral equation

c)

Green's integral equation

d)

Markov integral equation

(xxxiii)

The solution of Fredholm integral equation $y(x) + \int_0^1 e^{x-t} y(t)\, dt = 2xe^x$ is

a)

$$y(x) = e^x\left(1 - x^2\right)$$

b)

$$y(x) = e^x\left(2x - \frac{2}{3}\right)$$

c)

d) None of these

$$y(x) = e^x\left(x - \frac{2x}{3}\right)$$

(xxxiv)

The Laplace transformation of $\cos(at)u(t)$ is

a) $\frac{a}{a^2+s^2}$

b) $\frac{a^2}{a^2+s^2}$

c) $\frac{s}{a^2+s^2}$ |

d) $\frac{as}{a^2+s^2}$

(xxxv)

The solution of $y(x) = \cos x + \lambda\int_0^{\pi} \sin x \, y(t)\,dt$ is

a) $y(x) = \cos x,$ $provided \ \lambda \neq \frac{1}{2}$ |

b) $y(x) = \sin x,$ $provided \ \lambda \neq \frac{1}{2}$

c)

d) None of these

$$y(x) = \tan x,$$

$$provided\ \lambda \neq \frac{1}{2}$$

(xxxvi)

Let $f(x)$ be a periodic function with period L. Then which of the following is false

a)

b)

$$f(x+L) = f(x)$$

$$f(x+2L) = f(x)$$

c)

d) None of these

$$f(x+L) = f(x-L)$$

(xxxvii)

The solution of the integral equation $u(x) = 1 + \int_0^x u(t)dt$ (Volterra Integral equation of 2nd kind) is

a)

b)

$$e^{-x}$$

$$e^{x}$$

c)

d) None of these

$$e^{2x}$$

(xxxviii)

The initial value problem corresponding to the integral equation $y(x)=1+\int_0^x y(t)\,dt$

Is

a)

$$y' - y = 0, y(0) = 1$$

b)

$$y' + y = 0, y(0) = 0$$

c)

$$y' - y = 0, y(0) = 0$$

d)

$$y' + y = 0, y(0) = 1$$

(xxxix)

value of α for which the integral equation $u(x)=\alpha\int_0^1 e^{x-t}u(t)\,dx$,has a non –trivial solution is

a) -2

b) -1

c) 1

d) 2

(xl)

A function $f(t)$ is of exponential order if there exists real constant M and α such that

a)

$$|f(t)| \le Me^{\alpha}$$

b)

$$|f(t)| \le Me^{\alpha t}$$

c)

$$|f(t)| \le e^{\alpha} Mt$$

d)

Other

(xli)

Let $k(x,t)=\begin{cases} x+t, & 0\le t\le x \\ 0 & , otherwise \end{cases}$

Then , the integral equation $y(x)=1+\lambda\int_0^1 y(t)k(x,t)dt$, has

a) A unique solution for every value of λ

b) No solution for any value of λ

c) a unique solution for finitely many values of λ only

d) infinitely many solutions for finitely many values of λ

(xlii)

The initial value problem corresponding to the integral equation $y(x)=1+\int_0^x y(t)dt$ is

a) $y'-y=0, y(0)=1$

b) $y'+y=0, y(0)=0$

c) $y'-y=0, y(0)=0$

d) $y+y=0, y(0)=1$

(xliii)

$F_c[xf(x)]=$

a)

$$\frac{dF_s[f(x)]}{ds}$$

b)

$$\frac{dF_s[f(s)]}{dx}$$

c)

$$\frac{dF_x[f(s)]}{dx}$$

d) Other

(xliv)

A Volterra integral equation of second kind is of the form

a)

$$\phi(x)=\int_a^x K(x,t)\phi(t)\,dt$$

b)

$$\phi(x)=f(x)+\int_a^x K(x,t)\phi(t)\,dt$$

c)

$$\phi(x)=f(x)+\int_a^b K(x,t)\phi(t)\,dt$$

d)

$$\phi(x)=\int_a^b K(x,t)\phi(t)\,dt$$

(xlv)

The values of λ for which the integral equation $y(x)=\lambda\int_0^1(6x-t)\,y(t)dt$ has a non-trivial solution , are given by the roots of the equation

a)

$$(3\lambda-1)(2+\lambda)-\lambda^2=0$$

b)

$$(3\lambda-1)(2+\lambda)+2=0$$

c)

d)

$$(3\lambda-1)(2+\lambda)-4\lambda^2=0 \qquad (3\lambda-1)(2+\lambda)+\lambda^3=0$$

(xlvi)

The technique of transforming a particular differential equation into algebraic problem is called

a) dynamic

b) operational calculus

c) static

d) Integral

(xlvii)

For a given periodic function $f(t)=\begin{cases}2t & 0\le t\le 2\\ 4 & 2\le t\le 6\end{cases}$ with period 6. The Fourier coefficient b_1 is

a) -75.6800

b) 0.7468

c) -0.7468

d) None of these

(xlviii)

If a functional I[y(x)] having a variation attains a …………on $y = y_0(x)$ then at $y = y_0(x)$, $\delta(I) = 0$

a) maximum

b) minimum

c)

d) none of these

maximum or minimum

(xlix)

Consider $I = \int_0^{\pi} (y'^2 - y^2) dx$, with y(0)=0, y(1)=1, then inf.[y]=

a) 23/12
b) 21/24
c) 18/25
d) None of these

(l)

The extremal of the functional $\int_a^b \left(y + \frac{y^3}{3}\right) dx$ is

a) Maximum
b) Minimum
c) No solution
d) None of these

(li)

The extremal of $\int_0^2 (\frac{y'^2}{x}) dx$, y(0)=0 , y(2)=1 is

a) ellipse
b) circle
c) straight line
d) parabola

(lii)

Geodesics on a surface is

a) curves along which distance between two points on the surface is minimum

b) curves along which distance between two points on the surface is maximum

c) both the curves along which distance between two points on the surface is minimum & maximum

d) None of these

(liii)

A curve C of given length l which minimizes the curved surface area of solid generated by the revolution of C about x-axis is a

a) Sphere

b) Catenary

c) Ellipse

d) None of these

(liv)

The order of the differential equation is $F_y - \frac{d}{dx}F_{y'} + \frac{d^2}{dx^2}F_{y''} - \cdots + (-1)^n \frac{d^n}{dx^n}F_{y^n} = 0$

a) n

b) 2n

c) 3n

d) n+1

(lv)

The shortest distance between the point (0,0) and the straight line $x + y = \sqrt{2}$ is

a) 2

b) 1

c) 3

d) 4

(lvi)

Shortest distance between lines $L_1 = (x+1)/3 = (y+2)/1 = (z+1)/2$,

$L_2 = (x-2)/1 = (y+2)/2 = (z-3)/3$ is

a)

17/ ?3

b)

17/ 5?3

c)

17/ 5

d) other

(lvii)

The general form of a linear integral equation is

a)

$$y(x) = \int_a^b K(x,t)[y(t)]^2 dt$$

b)

$$g(x)y(x) = f(x) + \lambda \int_a K(x,t)y(t)dt$$

c)

$$\int_a^b K(x,t)y(t)dt = f(x)$$

d)

None of these

(lviii)

The resolvent kernel of the integral equation $y(x) = x - \int_0^x xt^2 y(t)dt, x > 0$ is

a)

$$xt^2 e^{-\frac{x^4-t^4}{4}}$$

b)

$$xt^2 e^{-\frac{x^2-t^2}{2}}$$

c)

$xte^{-\frac{x^4-t^4}{4}}$

d)

None of these

(lix)

Which of the following is a linear integral equation?

a)

$\int_a^b K(x,t)y(t)dt = f(x)$

b)

$y(x) - \lambda \int_a^b K(x,t)y(t)dt = f(x)$

c)

$\int_a^b K(x,t)y(t)dt = f(x)$

and $y(x) - \lambda \int_a^b K(x,t)y(t)dt = f(x)$

d)

$y(x) = \int_a^b K(x,t)[y(t)]^2 dt$

(lx)

In $g(x)y(x) = f(x) + \lambda \int_a^{\square} K(x,t)y(t)dt$, the upper limit

a)

may be the any value

b)

may be variable of x

c)

may be fixed constant

d)

Either may be variable of x or may be fixed constant