BRAINWARE UNIVERSITY
Term End Examination 2020 - 21
Programme – Master of Science in Mathematics
Course Name – Measure and Probability Theory
Course Code - MSCME302
Semester / Year - Semester III

Time allotted : 75 Minutes Full Marks : 60

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 60=60

1. *(Answer any Sixty)*

(i) <input type="image" src="/apttest/fck_image/q18.png" width="687" height="51" />

a) <input type="image" src="/apttest/fck_image/q18,1.png" width="110" height="31" />

b) $= \sum P(A_k)$

c) $\geq \sum P(A_k)$

d) Other

(ii) <input type="image" src="/apttest/fck_image/q3(1).png" width="502" height="62" />

a) infinitely many

b) all but finitely many

c) finitely many

d) all

(iii) The upper limit, limsup X_n of an increasing sequence $\{X_n\}$ of sets equals to

a) <input type="image" src="/apttest/fck_image/q5,1.png" width="58" height="50" />

b) <input type="image" src="/apttest/fck_image/q5,2.png" width="63" height="48" />

c)

Does not exists

d) Other

(iv)

Let ? is defined on an algebra of sets. Then ? is countably additive if for any finite (or countably infinite) system of (disjoint) sets X_n, we have

a) <input type="image" src="/apttest/fck_image/q19,2(1).png" width="173" height="31" />

b)

$$\mu\left(\sum X_n\right) = \sum \mu(X_n)$$

c)

$$\mu\left(\sum X_n\right) \leq \sum \mu(X_n)$$

d) Other

(v)

An extended realvalued, non-negative, countably additive function ? defined on a ring **R** of subsets of a set X with the value at empty set zero is called a ………….

a)

measurable function

b)

measure

c)

probability measure

d) None of these

(vi)

A countably additive set function is ………..

a)

b)

additive but not continuous

continuous but not additive

c)

both continuous and additive

d)

neither continuous nor additive

(vii)

A measure ? on an algebra **R** of subsets of a set X is called totally finite measure if

a)

?(X)=1

b)

?(X)=0

c)

?(X) is finite

d)

?(X) is infinity

(viii) <input type="image" src="/apttest/fck_image/q21.jpg" width="406" height="23" />

a) <input type="image" src="/apttest/fck_image/q5,2(5).png" width="63" height="48" />

b)

$$\bigcap_{n=1}^{\infty} X_n$$

c) <input type="image" src="/apttest/fck_image/q21,a(1).jpg" width="91" height="52" />

d) <input type="image" src="/apttest/fck_image/q22,b(2).jpg" width="91" height="55" />

(ix)

A measure is said to be complete if every subset of a measurable set of ………. is also measurable.

a) finite measure

b) infinite measure

c) zero measure

d) None of these

(x) <input type="image" src="/apttest/fck_image/q27.jpg" width="451" height="22" />

a) <input type="image" src="/apttest/fck_image/q27,a.jpg" width="82" height="26" />

b) <input type="image" src="/apttest/fck_image/q27,b.jpg" width="78" height="26" />

c) <input type="image" src="/apttest/fck_image/q27,c.jpg" width="197" height="25" />

d) Other

(xi)

Which of the following set is not a Borel set of **R** of the set of all real numbers?

a) open set

b) closed set

c) half open interval

d) Other

(xii) <input type="image" src="/apttest/fck_image/q30.jpg" width="517" height="45" />

a) finite subset of X only

b) all subset of X

c) measurable subset E of X

d) None of these

(xiii) <input type="image" src="/apttest/fck_image/q32(1).jpg" width="545" height="46" />

a) $\overline{\mu(S) \leq \mu(T)}$

b) $\overline{\mu(S) \geq \mu(T)}$

c) $\overline{\mu(S) = \mu(T)}$

d) Other

(xiv)

If $a \leq f(x) \leq b$ in E, then

a) $a\mu(E) \leq F(E) \leq b\mu(E)$

b) $a\mu(E) \geq F(E) \geq b\mu(E)$

c) $a\mu(E) = F(E) = b\mu(E)$

d) $a\mu(E) = F(E) = b\mu(E)$

(xv)

Three coins are tossed at random. Then the probability of the event of at least one head is

a) 3/8

b) 7/8

c) 2/9

d) 5/9

(xvi)

If F is a nondecreasing, right-continuous function satisfying…………, then there exists on some probability space a random variable X for which F(x) = P[X ? x].

a)

b)

$\lim_{x\to\infty} F(X) = 1$ $\overline{\lim_{x\to-\infty} F(X) = 0}$ $\lim_{x\to\infty} F(X) = 1$

c)

d) None of these

$\overline{\lim_{x\to-\infty} F(X) = 0}$

(xvii)

Borel-Cantelli lemma is about

a)

b)

Sums of events

Sequences of events

c)

d) Other

Independence of events

(xviii)

A sequence of random variables $\{X_n\}$ is convergent a. e. to X implies

a)

b)

$\lim P\{|X_n - X| \geq 1/2\} = 0$

$\lim P\{|X_n - X| \leq 1/2\} = 0$

c)

d)

$\lim P\{|X_n - X| > 1/2\} = 0$

$\lim P\{|X_n - X| < 1/2\} = 0$

(xix)

A sequence of random variables $\{X_n\}$ is convergent a. e. to X . Then for every $\varepsilon > 0$

a)

b)

$$\lim_{m\to\infty} P\{|X_n - X_{n'}| < \varepsilon, n' > n \geq m\} = 1$$

$$\lim_{m\to\infty} P\{|X_n - X_{n'}| < \varepsilon, n' > n \geq m\} = 0$$

c) $$\lim_{m\to\infty} P\{|X_n - X_{n'}| \geq \varepsilon, n' > n \geq m\} = 1$$

d) $$\lim_{m\to\infty} P\{|X_n - X_{n'}| > \varepsilon, n' > n \geq m\} = 0$$

(xx)

$X_n \to 0$ in probability

a) $$\text{if } E\left(\frac{|X_n|}{1+|X_n|}\right) \to 0$$

b) $$\text{if and only if } E\left(\frac{|X_n|}{1+|X_n|}\right) \to 0$$

c) $$\text{only if } E\left(\frac{|X_n|}{1+|X_n|}\right) \to 0$$

d) Other

(xxi)

Convergence in L^p implies that in

a) $L^r, r<p$

b) $L^r, r>p$

c)

d)

L^r,r is an integer

None of these

(xxii)

If $X_n \to X$ in L^p and $Y_n \to Y$ in $L^{\frac{p}{p-1}}$ then $X_n Y_n \to XY$ in

a)

L^p

b)

L^1

c)

$L^{\frac{p}{p-1}}$

d) Does not converge

(xxiii)

For each events E_n, we have

a)

$P(\limsup E_n) = \lim P\left(\bigcup_{n=m}^{\infty} E_n\right)$

b)

$P(\limsup E_n) = 1 - \lim P\left(\bigcup_{n=m}^{\infty} E_n\right)$

c)

$P(\limsup E_n) = \lim P\left(\bigcap_{n=m}^{\infty} E_n\right)$

d) Other

(xxiv)

A sequence of random variables $\{X_n\}$ is convergent a. e. to 0 if for every $\varepsilon > 0$

a) $P\{|X_n| < \varepsilon, i.o\} = 0$

b) $P\{|X_n| > \varepsilon, i.o\} = 0$

c) $P\{|X_n| < \varepsilon, i.o\} = 1$

d) Other

(xxv)

The value of $\{w : \lim_{n\to\infty} X_n(w) = 0\}$ equals to

a) $\bigcup_{m=1}^{\infty}\bigcap_{n=m}^{\infty}\{|X_n| > \frac{1}{m}\}$

b) $\bigcap_{m=1}^{\infty}\bigcup_{k=1}^{\infty}\bigcap_{n=k}^{\infty}\{|X_n| \le \frac{1}{m}\}$

c) $\bigcup_{m=1}^{\infty}\bigcap_{n=m}^{\infty}\{|X_n| > \frac{1}{n}\}$

d) $\bigcap_{m=1}^{\infty}\bigcup_{k=1}^{\infty}\bigcap_{n=k}^{\infty}\{|X_n| > \frac{1}{k}\}$

(xxvi)

If the events $\{E_n\}$ are independent, then

a) $P(E_n, i.o.) = 1$

b) $P(E_n, i.o.) = 0$

c) $P(E_n, i.o.) = \infty$

d) Other

(xxvii)

If the events $\{E_n\}$ are pair-wise independent, then the value of $P(\lim\sup E_n) = 1$ if

a)

$$\sum_n P(E_n) = \infty$$

b)

$$\sum_n P(E_n) < \infty$$

c)

$$\sum_n P(E_n) = 0$$

d) Other

(xxviii)

A sequence of random variables $\{x_n\}$ converges in probability to a random variable x. Then $\lim_{n\to\infty} p(|x_n - x| > 1/2)$ equals to

a) 0

b) 1/2

c) 1

d) Other

(xxix)

If $x_n \to x$ in probability then

a)

$$F_n(x_n) \to F(x)$$

b)

$$F_n(x) \to F(x)$$

c) <input type="image" src="/apttest/fck_image/q123,c.jpg" width="122" height="27" />

d) Other

(xxx)

If x_n has mean $\overline{x_n}$ and standard deviation σ_n then $x_n - \overline{x_n}$ converges to

a) $0 \text{ if } \sigma_n \to 0$

b) $1 \text{ if } \sigma_n \to 0$

c) 0

d) 1

(xxxi)

Let $y_k, k = 1,2,..$ be independent random variables with mean $\overline{y_k}, k = 1,2,..$ and standard deviation $\sigma_k, k = 1,2,..$ respectively. Let $x_n = \frac{1}{n}\sum_{k=1}^{n} y_k$, $m_n = \frac{1}{n}\sum_{k=1}^{n} \overline{y_k}$. Then $x_n - m_n \to 0$ in probability if

a) $\sum_{k=1}^{n} \sigma_k^{\ 2} = O(n)$

b) $\sum_{k=1}^{n} \sigma_k^{\ 2} = O(n^2)$

c) $\sum_{k=1}^{n} \sigma_k^{\ 2} \to 0$

d) Other

(xxxii)

Let θ be a irrational number then $\theta, 2\theta, 3\theta, ...$ is

a) Uniformly distributed modulo 1

b) Uniformly distributed module 0

c)

d) Other

Not uniformly distributed

(xxxiii)

A $K-L$ function is…… in every finite interval.

a) unbounded

b) bounded

c) continuous

d) Other

(xxxiv)

Let $\psi(t)$ be a K-L function with representation (a, G). Then G is…..in R

a) unbounded

b) bounded

c) continuous

d) None of these

(xxxv)

Let $\psi(t)$ be a K-L function with representation (a, G). Then G is bounded only in

a) the set of all positive real numbers

b) the set of all negative real numbers

c) the set of all real numbers

d) on the finite interval

(xxxvi)

Let $\psi_n(t)$ has K-L representation $(a_n.G_n)$ and $\psi_n(t) \to \psi(t), \forall t$. Then $\psi(t)$ have K-L representation $(a\ ,G)$ and

a) $a_n \to a$

b) $G_n \to G$

c) All the above

d) None of the above

(xxxvii)

If $X_n \Rightarrow X$ and X_n are uniformly integrable, then X is

a) always integrable

b) never uniformly integrable

c) may not be integrable

d) Other

(xxxviii)

$\mu_n(A) \to \mu(A), \forall A$, where A is a μ-continuity set if and only if $\int \mathrm{fd}\mu_n \to \int \mathrm{fd}\mu$, where f

a) bounded real functions

b) continuous real functions

c) bounded and continuous real functions

d) None of these

(xxxix)

If $X_n \Rightarrow X$ and $X_n - Y_n \Rightarrow 0$. Then

a) $X_n Y_n \Rightarrow X$

b) $X_n Y_n \Rightarrow 2X$

c) $Y_n \Rightarrow X$

d) Other

(xl)

For every sequence $\{F_n\}$ of distribution functions there exists a subsequence $\{F_{nk}\}$ and a non-decreasing, ……….. function F such that $\lim F_{n_k}(x) = F(x)$ at continuity points x of F.

a) non-increasing

b) non-decreasing

c) strictly-decreasing

d) strictly- increasing

(xli)

The variance of the random variable is

a)

b)

$$\int_X |x-\bar{x}| d\mu$$

$$\int_X |x-\bar{x}|^2 d\mu$$

c)

d) Other

$$\sqrt{\int_X |x-\bar{x}| d\mu}$$

(xlii)

Let X and Y be two random variables such that $Y = a + bX$ where a and b are two constants. Then $Var(Y)$ is

a)

b)

$$b^2 Var(X)$$

$$a^2 Var(X)$$

c)

d)

$$a + b^2 Var(X)$$

$$a^2 + b^2 Var(X)$$

(xliii)

Let $\alpha(x)$ be a non-negative function of the random variable x and $k > 0$ then

a)

b)

$$\frac{1}{k} E(\alpha(x)) \le P(\alpha(x) \ge k)$$

$$\frac{1}{k} E(\alpha(x)) \ge P(\alpha(x) \ge k)$$

c)

d)

$$E(\alpha(x)) \le P(\alpha(x) \ge k)$$

$$\frac{1}{k} E(\alpha(x)) \le P(\alpha(x) \le k)$$

(xliv)

Let $\alpha(x)$ be a non-negative function of the random variable x then

a) $E(\alpha(x)) \leq P(\alpha(x) \geq 1)$

b) $E(\alpha(x)) \geq P(\alpha(x) \geq 1)$

c) $E(\alpha(x)) = P(\alpha(x) \geq 1)$

d) Other

(xlv)

The mean of a Poisson distribution with parameter μ is

a) μ

b) μ^2

c) 2μ

d) Other

(xlvi)

If $(x_1, x_2, \ldots, x_n)$ is a real random vector, then

a) $E(x_1 + x_2 + \ldots + x_n) = E(x_1 . x_2 \ldots x_n)$

b) $E(x_1 + x_2 + \ldots + x_n) = E(x_1) + E(x_2) + \ldots + E(x_n)$

c)

d)

$E(x_1 + x_2 + \ldots + x_n)$
$\leq E(x_1.x_2 \ldots x_n)$

$E(x_1 + x_2 + \ldots + x_n)$
$\leq E(x_1) + E(x_2) + \ldots + E(x_n)$

(xlvii)

Let $\alpha(x_1, x_2)$ be a function of two independent variable x_1, x_2 and $X = X_1 \times X_2$. Then

a)

$$\int_X \alpha(x_1, x_2) d\mu = \iint_X \alpha(x_1, x_2) d\mu^2$$

b)

$$\int_X \alpha(x_1, x_2) d\mu = \iint_X \alpha(x_1, x_2) d\mu_1 d\mu_2$$

c)

$$\int_X \alpha(x_1, x_2) d\mu = 0$$

d) Other

(xlviii) The sum of independent variables with binomial distributions

a) may not be a binomial distribution
b) must be a binomial distribution
c) never a binomial distribution
d) None of these

(xlix) The sum of independent variables of normal distributions is

a) may not be a normal distribution
b) must be a normal distribution
c) never a normal distribution
d) None of these

(l) The sum of two independent variables has Poisson distributions then

a) At-least one has Poisson distribution
b) Both variables have Poisson distribution
c) . At-most one variable has Poisson distribution
d) Both variables may not have Poisson distribution

(li)

Let x_1, x_2 be two independent variables with their distribution function F_1, F_2 respectively then their sum has the distribution function F given by the formula

a)

$$F(x) = F_1(x) + F_2(x)$$

b)

$$F(x) = F_1(x)F_2(x)$$

c)

$$F(x) = F^2{}_1(x)F_2(x)$$

d) Other

(lii)

The characteristic function $\varphi(t)$ of a distribution function $F(x)$ satisfies

a)

$$\varphi(t) = 1$$

b)

$$|\varphi(t)| = 1$$

c)

$$|\varphi(t)| \leq 1$$

d)

$$|\varphi(t)| \geq 1$$

(liii)

The characteristic function $\varphi(t)$ of a distribution function $F(x)$ satisfies

a)

$$\varphi(0) = 1$$

b)

$$\varphi(t) = 1$$

c)

$$\varphi(t) = 0$$

d)

$$|\varphi(t)| = 1$$

(liv)

Let A be a support of a distribution m. Then m(A) is equal to

a) 0
b) 1
c) Non conclusion can be done
d) Other

(lv)

The sum of independent variables with Poisson distributions c_1, c_2 is Poisson and has parameter …..

a) $c_1 . c_2$
b) $(c_1 .+ c_2)^2$
c) c_1+c_2
d) None of these

(lvi)

Let A and B are two m-measurable sets and A is a subset of B. If m(B)=0, then m(A)

a) 1
b) 0
c) less than 0
d) Other

(lvii) The mean of a Poisson distribution with parameter 10 is

a) 10
b) -10
c) 100
d) -100

(lviii)

Which of the following is a ring of subsets of X={1,2,3}?

a) {O, X, {1}, {2}}
b) {O, X, {1}, {2}, {1, 2}}
c) {O, X}
d) Other

(lix)

Let X be a finite set. Then the class of all finite subsets of X is not a/an

a) ring
b) algebra
c) algebra but a ring
d) Other

(lx) <div>An extended real valued set function m defined on a class E of sets is additive if for any two disjoint sets A and B belong to the class, we have:</div>

a) m(AUB)<m(A)+m(B)
b) m(AUB)>m(A)+m(B)
c) m(AUB)=m(A)+m(B)
d) Other