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BRAINWARE UNIVERSITY

Term End Examination 2025-2026
Programme – M.Sc.(MATH)-2025
Course Name – Linear Algebra
Course Code - MSM10401
(Semester I)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) If a linear transformation is represented by matrix A, then T(v)=

a) A+v
b) Av
c) A^Tv
d) vA

(ii) Select the correct option: An annihilating polynomial of a linear operator is

a) A polynomial that annihilates identity
b) p(T) = 0
c) A multiple of characteristic polynomial
d) None of these

(iii) $S=\left\{\begin{pmatrix} x_1 & 0 \\ 0 & x_2 \end{pmatrix} \mid x_1, x_2 \in R\right\}$, then identify the value of dim(S)

a) 2
b) 3
c) 5
d) None of these

(iv) Identify the number of vectors in the basis of the vector space $\left\{\begin{bmatrix} x & 0 \\ x & y \end{bmatrix} : x, y \in R\right\}$

a) 1
b) 2
c) 3
d) 0

(v) Identify the true definition of a symmetric matrix from the options below:

a) A square matrix A is said to be symmetric if $A=-A$
b) A square matrix A is said to be symmetric if $A=-A'$
c) A square matrix A is said to be symmetric if $A=A'$
d) A square matrix A is said to be symmetric if $A=\left[adj(A)\right]'$

(vi) Let A and B be matrices of the same order. Identify the property that is always true.

a) $(A-B)(A+B)=A^2-B^2$
b) $(A+B)^2=A^2+2AB+B^2$
c) $A(A+B)=A^2+AB$
d) None of these

(vii) If $V=R^3$ be equipped with inner product $(x,y)=x_1y_1+2x_2y_2+3x_3y_3$, In this inner product space $(V,(.,.))$ Choose the pairs of orthonormal vector

a) $u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

b) $u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$

c) $u = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ 0 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

d) $u = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, v = \begin{bmatrix} 0 \\ \frac{1}{2} \\ 0 \end{bmatrix}$

(viii) Let W_1 and W_2 be subspaces of a vector space V having dimensions m and n, respectively, where $m > n$, then choose the right option.

a) $\dim(W_1 \cap W_2) = n$

b) $\dim(W_1 \cap W_2) = m$

c) $\dim(W_1 \cap W_2) \leq n$

d) $\dim(W_1 \cap W_2) \leq m$

(ix) Define T: $R^2 \to R^2$ by $T(a_1, a_2) = (a_1, -a_2)$. Then choose the right option

a) T is called the reflection about the y -axis

b) T is called the reflection about the x -axis

c) T is called the projection on the x -axis

d) T is called the projection on the y –axis

(x)

Let $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$, then calculate $dim\left(span\left(\left\{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right\}\right)\right)$

a) 1

b) 2

c) 3

d) 0

(xi)

Let $\alpha = e^{\frac{2\pi i}{3}}$ and the matrix $M = \begin{pmatrix} 1 & \alpha & \alpha^2 \\ 0 & \alpha & \alpha^2 \\ 0 & 0 & \alpha^2 \end{pmatrix}$. Evaluate the trace of the matrix

$I + M + M^2$

a) -3

b) 0

c) 1

d) 3

(xii) Choose the correct option: The system of equations 4x - 2y = 5, - 6x + 3y = 1 has

a) Single solution

b) Finite number of solutions

c) Infinite number of solutions

d) No solution

(xiii) If a square matrix of order 10 has exactly 5 distinct eigen values, then conclude the degree of the minimal polynomial.

a) at least 5

b) at most 5

c) always 5

d) exactly 10

(xiv) Select the right option: The matrix $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ is

a) indefinite

b) positive semidefinite

c) positive definite

d) None of these

(xv) The leading principal minors of a 4 by 4 matrix are $|A1| = -1$, $|A2| = 1$, $|A3| = 2$, and $|A4| = |A| = 0$. Then select the correct statement

a) the matrix is negative semidefinite

b) its definiteness cannot be determined with this information

c) the matrix is indefinite

d) None of these

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Identify the eigenvalues of $\begin{bmatrix} 0 & -i \\ i & 1 \end{bmatrix}$. (3)

3. Show that every complex square matrix A is similar to its transpose. (3)

4. Apply Cayley-Hamilton theorem to find A^{-1}, where $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$. (3)

5. Suppose two or more nonzero vectors $v_1, v_2, \ldots v_m$ are linearly dependent then justify that one of them is a linear combination of the preceding vectors. (3)

6. Illustrate if A is the companion matrix of a monic polynomial p, then p is both minimal and the characteristic polynomial of A. (3)

OR

Explain orthogonal complement and orthogonal projection of V on W. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. State and prove Cauchy-Schwarz inequality. (5)

8. If α and β be any two vectors in an inner product space $(R^2, \|.,.\|)$, then illustrate that $\|\alpha+\beta\| \le \|\alpha\| + \|\beta\|$. (5)

9. Evaluate that the set of vectors $\{(3,0,4),(-4,0,3),(0,9,0)\}$ is orthogonal, while the set $\left\{\frac{1}{5}(3,0,4),\frac{1}{5}(-4,0,3),(0,1,0)\right\}$ is orthonormal (5)

10. Establish that if a square matrix A has a left inverse B and a right inverse C then B=C. (5)

11. Justify the following statement: (5)

"Eigenvectors of a linear operator T are also generalized eigenvectors of T".

12. Justify that the set of vectors {(1,2,2), (2,-2,1), (2,1,-2)} is an orthogonal basis of the Euclidean space $\mathbb{R}^3$ with standard inner product. Express (4,3,2) as a linear combination of these basis vectors. (5)

OR

Use Gaussian elimination to construct the solution for the given system of equations: $3x+y-z=1,\ x-y+z=-3,\ 2x+y+z=0$. (5)

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