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Barasat, Kolkata -700125

Term End Examination 2025-2026
Programme – M.Sc.(MATH)-2025
Course Name – Mathematical Statistics
Course Code - MSM10404B
(Semester I)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Select the example of qualitative data.

a) Age of a person b) Gender of a person
c) Height of a tree d) Number of books

(ii) Select the example of quantitative data.

a) Religion b) Temperature
c) Occupation d) Eye color

(iii) Select the measure that quantifies the strength and direction of linear association between two variables.

a) Covariance b) Pearson correlation coefficient
c) Mode d) Regression coefficient

(iv) Select the range of Pearson's r.

a) 0 to 1 b) -1 to $+1$
c) -1 to 0 d) None of these

(v) Correlation measures _____. Select the correct option.

a) Causation b) Variance
c) Association d) Error

(vi) Select the theorem that says that the sampling distribution of the sample mean becomes approximately normal as sample size increases.

a) Law of Large Numbers b) Chebyshev's Theorem
c) Central Limit Theorem d) Slutsky's Theorem

(vii) The standard error of the mean decreases when _____. Select the correct option.

a) The sample size decreases b) The sample size increases
c) The population variance increases d) The data are skewed

(viii) Choose the Maximum Likelihood Estimator of the parameter λ for Poisson distribution.

a) Sample mean
b) Sample median
c) Sample standard deviation
d) None of the alternatives

(ix) In a normal distribution with known variance, choose the MLE of the mean.

a) Median
b) Mode
c) Sample mean
d) Sample variance

(x) For a parameter θ, the Rao-Blackwell theorem provides a method to ____. Choose the correct option.

a) Increase bias
b) Reduce power
c) Improve an unbiased estimator
d) Construct a confidence interval

(xi) The Cramer-Rao lower bound gives ____. Choose the correct option.

a) Maximum variance of estimator
b) Lower bound of variance of unbiased estimator
c) Sample size
d) Confidence level

(xii) The method of moments equates sample moments to ____. Choose the correct option.

a) Mode
b) Population moments
c) Likelihood values
d) Posterior probabilities

(xiii) For known variance, the confidence interval for mean is based on ____. Choose the correct option.

a) F-distribution
b) Chi-square distribution
c) Standard normal distribution
d) Uniform distribution

(xiv) The MVUE is unique if ____. Choose the correct option.

a) It exists
b) The sample size is large
c) Variance is unknown
d) The estimator is consistent

(xv) In one-way ANOVA, if F calculated > F critical, then choose the decision.

a) Reject null hypothesis
b) Accept null hypothesis
c) Increase sample size
d) Conduct chi-square test

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Group-B

(Short Answer Type Questions) 3 x 5=15

2. Describe the PDF of the minimum of order statistics of uniform distribution. (3)

3. If $X \sim N_3(\mu, \Sigma), \mu = \begin{pmatrix} -3 \\ 1 \\ 4 \end{pmatrix}, \Sigma = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}$, show that (X_1, X_2) and X_3 are independent. (3)

4. Determine the sufficient estimator of the parameter p for Bernoulli distribution. (3)

5. In a survey of 500 students, the number of students in various GPA intervals is recorded. Explain how this data should be classified and why it is effective for analysis. (3)

6. Consider a random sample having probability density function $f(x) = \frac{1}{\theta}, 0 < x < \theta$. Evaluate the MLE of θ. (3)

OR

Consider a random sample having probability density function $f(x) = \theta x^{\theta-1}, 0 < x < 1, \theta > 0$. Evaluate the sufficient estimator of θ. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Describe the PDF of the minimum and the maximum of order statistics of exponential distribution with parameter λ. (5)

8. Illustrate the following data using a frequency polygon: (5)

Marks in Mathematics	40-45	45-49	50-54	55-59	60-64	65-69
No. of students	1	3	2	4	5	6
Marks in Mathematics	70-74	75-79	80-84	85-89	90-95	95-99
No. of students	10	8	5	6	2	1

9. A random sample of size 36 is drawn from a normal population with mean 50 and standard deviation 12. Estimate the standard error of the mean. (5)

10. Consider a random sample having probability density function $f(x) = \frac{1}{\theta}e^{-x/\theta}$. Illustrate that the sample mean is Minimum Variance Bound Estimator of θ. (5)

11. Consider a random sample having probability density function $f(x) = e^{-(x-\theta)}, 0 \leq x < \infty$. Evaluate the MLE of θ. (5)

12. Consider a random sample having probability density function $f(x) = \frac{\beta^\alpha}{\Gamma\alpha} x^{\alpha-1} e^{-\beta x}, 0 \leq x < \infty$. If $\mu_r' = \frac{\Gamma(\alpha+r)}{\Gamma\alpha\, \beta^r}$, evaluate moment estimates of α and β. (5)

OR

Consider a random sample $x_1, x_2, \dots, x_n$ drawn from a normal distribution with parameters μ and $\sigma^2 = 4$. The critical region is given as $W = \{x_1 + 2x_2 + 3x_3 + \cdots + 10x_{10} \geq 0\}$. The hypotheses are $H_0: \mu = -1, H_1: \mu = 1$. Assess if it is an unbiased test. Given: $P(z \geq 1.4015) = 0.0808, P(z \geq -1.4015) = 0.9192$. (5)

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