14950

BRAINWARE UNIVERSITY

Term End Examination 2025-2026
Programme – M.Sc.(MATH)-2024
Course Name – Functional Analysis
Course Code - MSCMC301
(Semester III)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Select the correct answer. The value of $\|x\|$, where $x=(1,-1,1,-1,\ldots)\in l_\infty$ under supremum norm

a) -1
b) 0
c) 1
d) 2

(ii) Select the correct answer. A normed space is a Banach Space if and only if

a) Every sequence is convergent
b) Every Cauchy sequence is convergent
c) Every convergent sequence is Cauchy
d) None of these

(iii) Let d be a metric induced by a norm on a normed space X. Select the correct answer which is always true.

a) $d(-2x,-2y)\le 2d(x,y)$
b) $d(-2x,-2y)\le -2d(x,y)$
c) $d(-2x,-2y)=2d(x,y)$
d) $d(-2x,-2y)+2d(x,y)=0$

(iv) Select the correct answer. Let X be a n-dimensional normed space. Then the dimension of a proper subspace of X is always

a) n
b) less than or equal to n
c) less than n
d) greater than n

(v) Select the correct answer. Let $T:X\to Y$ be an unbounded linear operator between normed spaces X and Y. Then T is

a) always continuous
b) never continuous
c) Continuous at some points of X
d) T can't be unbounded

(vi) Select the correct answer. Let $T : X \to Y$ be a bounded linear operator between normed spaces X and Y. Then the null space of T is

a) closed
b) open
c) both closed and open
d) None of these

(vii) Select the correct answer. A mapping between two metric spaces from X to Y is called a closed map if:

a) Image of every closed set of X is closed in Y
b) Image of every subsets of X is closed in Y
c) Pre-image of every closed set of Y is open in X
d) Pre-image of every subsets of Y is closed in X

(viii) Write the correct answer. A functional is an operator whose range lies on

a) C
b) R
c) Either C or R
d) Any complete NLS

(ix) Let f be a continuous linear functional on an NLS X. Then choose the correct answer which is always true.

a) $\|f(x)\| \leq \|x\|$
b) $\|f(x)\| \leq \|f\|\|x\|$
c) $\|f(x)\| \geq \|x\|$
d) $\|f(x)\| \geq \|f\|\|x\|$

(x) Write the correct answer. Let a vector space X is not algebraically reflexive then X is

a) Finite dimensional
b) Infinite dimensional
c) Equal to X*
d) Other

(xi) Select the correct answer. If a Hilbert space X has a basis. Then X is

a) Separable metric space
b) Inseparable metric space
c) Both a and b can be possible
d) Either a or b always true

(xii) Select the correct answer. Let E be a subset of a separable inner product space. Then E is

a) separable
b) separable if E is a subspace
c) separable if E is a closed subspace
d) None of these

(xiii) Select the correct answer. An orthonormal set M in a Hilbert space H is total in H if

a) for all x in H the Parseval relation holds
b) for at-least x in H the Parseval relation holds
c) for all x in M the Parseval relation holds
d) for all x in $M^{\perp}$ the Parseval relation holds

(xiv) Select the correct answer. Let H be a Hilbert space contains a total orthonormal sequence then H is

a) inseparable
b) separable
c) all dense set are uncountable
d) other

(xv) Select the correct option. Application of Banach fixed point theorem to differential equations is valid when

a) The associated operator is a contraction
b) The differential equation is linear
c) The domain is bounded
d) The equation has constant coefficients

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Establish the continuity property of inner product space. (3)

3. State the condition under which a sequence in a normed space is Cauchy. (3)

4. Let $S = \left\{(-1)^m + \frac{1}{n} : m \in N, n \in N\right\}$. Analyze that 1 and -1 are limit points of S. (3)

5. Show that the mapping $f: (X, ||.||) \to \mathbb{R}$, defined by $f(x) = ||x||, \forall\, x \in X$ is continuous. (3)

6. Let T be a linear operator. Then justify that the range space is a vector space. (3)

OR

Justify that the inverse operator T^{-1} of a linear operator of T is a linear operator. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Explain that all matrices representing a given linear operator $T: X \to X$ on a finite-dimensional normed space X relative to various bases for X have the same eigenvalues. (5)

8. Let $T: X \to Y$ be a linear operator on X. Show that if T is continuous at some point on X then T is continuous on X. (5)

9. Explain the following statement.
If $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in X, then $\{x_n + y_n\}$ is a Cauchy sequence in X. (5)

10. Let Y be any closed subspace of a Hilbert space. Then justify that $H = Y \oplus Y^{\perp\perp}$. (5)

11. Let X and Y be any two normed linear spaces over the same scalar field. Explain that if X has finite dimension, then every linear operator $T: X \to Y$ is continuous. (5)

12. Justify that a norm linear space $X \neq \{0\}$ is a Banach space if the unit sphere $S[0,1] = \{x \in X: ||x|| = 1\}$ is complete. (5)

OR

Let $T: X \to Y$ be a linear operator on X. Justify that if T is a bounded linear operator, then the image T(S) of the unit sphere $S = \{x \in X: ||x|| = 1\}$ is bounded. (5)

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