14951

BRAINWARE UNIVERSITY

Term End Examination 2025-2026
Programme – M.Sc.(MATH)-2024
Course Name – Differential Geometry
Course Code - MSCMC302
(Semester III)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Identify the rectangular Cartesian coordinates of the point whose cylindrical coordinates are $\left(2,\frac{\pi}{3},1\right)$

a) $\left(1,\sqrt{3},1\right)$
b) $(1,2,1)$
c) $(1,3,1)$
d) None of these

(ii) Identify the rectangular Cartesian coordinates of the point whose cylindrical coordinates are $\left(6,\frac{\pi}{4},1\right)$

a) $\left(1,\sqrt{3},1\right)$
b) $\left(\frac{6}{\sqrt{2}},\frac{6}{\sqrt{2}},1\right)$
c) $(1,3,1)$
d) None of these

(iii) Evaluate the condition of a curve that the curvature of a curve be maximum.

a) When the curve is a straight line.
b) When the curve is a circle with a large radius.
c) When the curve is a circle with a small radius.
d) When the curve is a parabola.

(iv) Select the correct option. The tangent vector to a curve at a given point represents:

a) The direction of motion along the curve at that point.
b) The curvature of the curve at that point.
c) The perpendicular line to the curve at that point.
d) The rate of change of the curve's slope at that point.

(v) Select the correct option. The torsion of a curve measures

a) How fast the curve is changing direction
b) The rate of change of the curve's tangent vector
c) The perpendicular distance between the curve and a fixed point
d) The rate of change of the curve's principal normal vector

(vi) Select the correct option. Differential forms on surfaces are:

a) Scalars that depend on the position and direction on the surface
b) Vectors that depend only on the position on the surface
c) One-dimensional objects defined on the surface
d) Tensors that describe the curvature of the surface

(vii) Select the correct option. The second fundamental form of a surface describes:

a) The rate of change of the tangent vector along a curve
b) The rate of change of the normal vector along a curve
c) The curvatures of curves on the surface
d) The derivatives of the tangent vector

(viii) Select the correct option. For a flat surface, the Gaussian curvature is:

a) Always positive
b) Always negative
c) Always zero
d) Can be positive, negative, or zero depending on the surface's shape

(ix) Determine the kronecker delta $\delta^i{}_j$

a) Symmetric in i and j
b) Skew-symmetric in i and j
c) 1
d) None of these

(x) If A_m is a covariant vector , determine $\frac{\partial A_m}{\partial x^s} - \frac{\partial A_s}{\partial x^m}$

a) A tensor of type (0,2)
b) A tensor of type (1,3)
c) A tensor of type(0,1)
d) A tensor of type (1,1)

(xi) The coefficient g_{ij} defined in $ds^2 = g_{mn}dx^m dx^n$ then identify the components

a) Skew-symmetric covariant tensor of type (0,2)
b) Symmetric covariant tensor of type (0,2)
c) Skew-symmetric covariant tensor of type (1,2)
d) Symmetric covariant tensor of type (1,2)

(xii) If $g_{ij} = 0$ for $i \neq j$, then $[i\,i, k] =$ ________________ (Identify the correct option)

a) $\frac{1}{2}\cdot\frac{\partial g_{ii}}{\partial x^i}$
b) $-\frac{1}{2g_{ii}}\cdot\frac{\partial g_{jj}}{\partial x^i}$
c) $-\frac{1}{2}\cdot\frac{\partial g_{ii}}{\partial x^k}$
d) None of these

(xiii) A space curve is a plane curve if and only if its torsion is (Select the correct option)

a) 1
b) 0
c) Constant
d) None of these

(xiv) If the curves have the same principal normal then they are named as

a) Bertrand mates
b) Associates
c) No specific name
d) None of these

(xv) For the paraboloid $r = (u, v, u^2 - v^2)$, the ds is illustrate as

a) $ds^2 = (du)^2 + (u^2+1)(dv)^2$
b) $ds^2 = (du)^2 + (u^2+v^2)(dv)^2$
c) $ds^2 = (1+f_1^2)(du)^2 + u^2(dv)^2$
d) $ds^2 = (1+4u^2)(du)^2 - 4uvdudv + (1+4v^2)(dv)^2$

Group-B
(Short Answer Type Questions) 3 x 5=15

2. Evaluate the equation of the tangent to the curve $y^2 - yx^2 - 2x^5 = 0$ at the point (1,-1). (3)

3. Describe an example of a surface with constant negative Gaussian curvature. (3)

4. In terms of Cartesian parametric representation, show that $s = \int_a^u \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\, du$. (3)

5. Illustrate Gaussian curvature. (3)

6. Write Codazzi-Mainardi equation. (3)

OR

Write the differential of arc length for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Describe what do you mean by self-intersection point of a curve. Describe what do you mean by simple closed curve in $\mathbb{R}^2$. (5)

8. Evaluate the curvature of the curve $r = \{a(1 + \cos u), a \sin u, 2a \sin \frac{1}{2} u\}$. (5)

9. If $a_j^i b_p^j = c_p^i$, then justify that $(a_j^i)(b_p^j) = (c_p^i)$ and $|a_j^i||b_p^j| = |c_p^i|$. (5)

10. Define binormal of a curve. (5)

11. If a_{ij} is a skew-symmetric tensor, then justify that $(\delta_l^i \delta_l^k + \delta_l^i \delta_j^k) a_{ik} = 0$. (5)

12. Evaluate the indicatrix of the tangent of the helix γ given by r = (a cos u, a sin u, bu). (5)

OR

If $B_{ij} = A_{ji}$, where A_{ij} is a covariant tensor, then justify that B_{ij} is a tensor of order 2. (5)

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