17320

BRAINWARE UNIVERSITY

Term End Examination 2025-2026
Programme – B.Tech.(RA)-2022/B.Tech.(RA)-2023
Course Name – Probability Theory and Stochastic Process
Course Code - BSCR501
(Semester V)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Let X be a random variable with mean μ and variance σ^2 and k be any real number. Select the statement that represents Markov's inequality.

a) $P\{|X-\mu| \geq k\sigma\} \leq \frac{1}{k^2}$ b) $P\{|X-\mu| < k\sigma\} \geq 1-\frac{1}{k^2}$

c) $P\{|X| \geq k\} \leq \frac{E|X|}{k}$ d) None of these

(ii) The condition for mutual exclusiveness of two events A and B is _____. Select the correct option.

a) $P(A \cup B) = P(A) + P(B)$ b) $P(A+B) = P(A)P(B)$

c) $P(A-B) = P(A)P(B)$ d) $P(A \cap B) = P(A)P(B|A)$

(iii) The probability of any event A satisfies _____. Select the correct option.

a) $P(A) \geq 1$ b) $P(A) < 0$

c) $0 \leq P(A) \leq 1$ d) None of these

(iv) Let F(x) be the distribution function of the continuous random variable X. Choose the expression that describes the probability $P(a \leq X \leq b)$.

a) F(b)-F(a) b) F(b)+F(a)

c) F(a)-F(b) d) F(a) F(b)

(v) The moment generating function of a random variable X is defined as ____. Choose the correct answer.

a) $E(e^{tX})$ b) $E(e^{itX})$

c) E(tX) d) None of these

(vi) The mean of the binomial distribution is _____. Compute the correct option.

a) Less than the variance b) Equal to the variance

c) Greater than the variance d) None of these

(vii) Choose the process if the future value of a sample function cannot be predicted based on its past values.

a) Deterministic process b) Non-deterministic process

c) Independent process d) Statistical process

(viii) $R_{xx}(\tau)$= _____. Choose the correct option.

a) $E[X^2(t)]$ b) $\int_{-\infty}^{\infty} X(t)dt$

c) $\int_{-\infty}^{\infty} X^2(t)dt$ d) $E[X(t)X(t+\tau)]$

(ix) Priority queue discipline may be classified as _____. Choose the correct option.

a) Finite or infinite b) Limited or unlimited

c) Pre-emptive or non-pre-emptive d) All of the above

(x) On a certain day, number of customers in a system are 20, and number of customers in service are 12, select the number of customers waiting for service.

a) 10 b) 8

c) 20 d) None of these

(xi) Stochastic processes are _____. Select the correct option.

a) Random in nature b) Are function of time

c) Random in nature and are a function of time d) None of the mentioned

(xii) The variance of a random process $X(t)$ is defined as _____. Select the correct option.

a) $E[X^2(t)]$ b) $E[X^2(t)] - [E\{X(t)\}]^2$

c) $\int_{-\infty}^{\infty} X^2(t)dt$ d) $E[X(t)X(t+\tau)]$

(xiii) While calculating equilibrium probabilities for a Markov process, select the correct assumption.

a) there is a single absorbing state b) transition probabilities do not change

c) there is a single non-absorbing state d) none of these

(xiv) Markov analysis is useful for _____. Select the correct option.

a) Predicting the state of the system at some future time b) calculating transition probabilities at some future time

c) Predicting the steadiness of the system at some future time d) All of the above

(xv) In a matrix of transition probability, the element a_{ij}, where i=j is a _____. Select the correct option.

a) Gain
b) Loss
c) Retention
d) None of the above

Group-B
(Short Answer Type Questions) 3 x 5=15

2. Describe Jensen's inequality. (3)

3. The variable X follows normal distribution and has mean 140 and standard deviation 2. Estimate the probability that the value of X will lie between 138 and 143, given that $\phi(1.5) = 0.9332, \phi(-1) = 0.1587$. (3)

4. Consider the Markov chain having transition probability matrix (3)

$$P = \begin{bmatrix} 1/4 & 1/2 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

Compute $P(X_4 = 3|X_3 = 2), P(X_3 = 1|X_2 = 1)$.

5. Let $\{X_n, n \geq 0\}$ be a sequence of i.i.d. random variables with mean 0 and variance 1. Establish that it is a Weak Sense Stationary Process. (3)

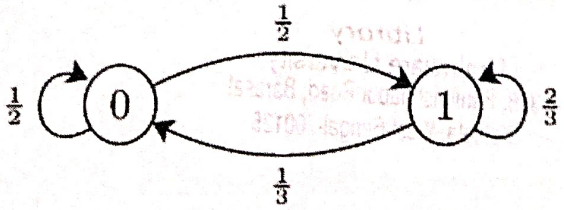
6. Consider the following matrix (3)

$$\begin{pmatrix} 0 & 1 \\ 0.5 & 0.5 \end{pmatrix}.$$

Justify if the matrix is a valid Transition Probability Matrix (TPM).

OR

Consider the Markov Chain shown in Figure. (3)



Assess if this chain is aperiodic.

Group-C
(Long Answer Type Questions) 5 x 6=30

7. Compute the steady-state probabilities of the Markov Chain having one-step TPM (5)

$$P^{(1)} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \end{pmatrix}.$$

8. An Economics consulting firm has created a model to predict recessions. The model predicts a recession with probability 80% when a recession is indeed coming and with probability 10% when no recession is coming. The unconditional probability of falling into a recession is 20%. If the model predicts a recession, compute the probability that a recession will indeed come. (5)

9. Two dice are thrown. Let A be the event that the sum of the points on the faces shown is odd, and B be the event of at least one face showing 1. Describe the complete sample space, the events A, B, $A \cap B$. Compute $P(A|B)$ and $P(B|A)$. (5)

10. Summarize the concept of splitting two independent Poisson process. (5)

11. Explain the memoryless property of the exponential distribution. (5)

12. In a M/M/1 queueing model with $E[s] = 0.22, \rho = 0.88$, evaluate L, L_q, W, W_q, the notations having their usual meaning. (5)

OR

Summarize birth and death process. (5)

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