



# BRAINWARE UNIVERSITY

Term End Examination 2022

Programme – MCA-2018/MCA-2020/MCA-2021

Course Name – Discrete Mathematics

Course Code - MMAT010102/MCA104

( Semester I )

Full Marks : 60

Time : 2:30 Hours

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

## Group-A

(Multiple Choice Type Question)

1 x 15=15

1. Choose the correct alternative from the following :

- (i) Let  $R$  be the real line consider the following subsets of the plane  $R \times R$ ,  
 $S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$ ,  $T = \{(x, y) : x - y \text{ is an integer}\}$ , Select the true statement?
- a)  $T$  is an equivalence relation on  $R$  but  $S$  is not.      b) Neither  $S$  nor  $T$  is an equivalence relation on  $R$   
c) Both  $S$  and  $T$  are equivalence relations on  $R$       d)  $S$  is an equivalence relation on  $R$  but  $T$  is not
- (ii) Choose the correct answer: A graph with no circuit and no parallel edges is called  
a) Multi graph      b) Pseudo graph  
c) Simple graph      d) None of these
- (iii) If a graph has 6 vertices and 15 edges then write down the size of its adjacency matrix is  
a)  $6 \times 6$       b)  $6 \times 15$   
c)  $15 \times 6$       d)  $15 \times 15$
- (iv) Choose the correct option: Number of edges in a complete graph with  $n$ -vertices is:  
a)  ${}^n C_1$       b)  ${}^n C_2$   
c)  ${}^n C_3$       d)  ${}^n C_n$
- (v) Choose the correct option: The root of a binary tree is the vertex having degree  
a) 1      b) 2  
c) 3      d) 4
- (vi) Let  $f : (-1, 1) \rightarrow B$  be a function defined by  $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$  and  $f$  is both one-one and onto, then select which one is set  $B$

a)  $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

b)  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

c)  $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

d)  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(vii)

Let  $f(x) = \sec x + \tan x$  and  $g(x) = \frac{\tan x}{1 - \sec x}$

(i)  $g$  is odd function

(ii)  $f$  is neither an odd function nor an even function

Select the right option.

a) (i) is true

b) (ii) is true

c) (i) and (ii) both are true

d) (i) and (ii) both are false

(viii) The set of all real numbers under usual addition formed a group. Then Write down the inverse of 2.36

a) 2.36

b) -2.36

c) 2.40

d) -2.40

(ix) Choose the correct option: A tree always is a

a) self-complement graph

b) Euler graph

c) simple graph

d) Hamiltonian graph

(x) Identify the least number of people 4 of whom will have same birthday of the week

a) 18

b) 42

c) 28

d) 22

(xi) Select the number of ways in which 6 different flowers can be arranged in a garland

a) 120

b) 60

c) 240

d) None of these

(xii) Choose the right option: The number of words of 5 different letters that can be formed by taking 2 letters from the word BOX and 3 letters from the word TABLE is

a) 120

b) 30

c) 3600

d) None of these

(xiii) If  $f: A \rightarrow \mathbb{R}$  where  $B = \{0, 1, 4, 9\}$  and  $f$  is defined by the rule  $f(x) = x^2$ . Identify for which set  $A$  is the one-to-one function

a)  $\{-1, 0, 1, 2\}$

b)  $\{-3, -1, 0, 2, 3\}$

c)  $\{-2, 0, -1, 2\}$

d)  $\{0, 1, 2, 3\}$

(xiv) Identify the right option: An one-to-one function is known as

a) injective function

b) surjective function

c) bijective function

d) none of these

(xv) Let  $p$ : It is cold and  $q$ : It is raining, then identify the symbolic form of the statement 'It is cold or it is not raining' is

a)  $p \vee q$

b)  $\neg p \vee q$

c)  $\neg p \wedge q$

d)  $p \vee \neg q$

### Group-B

(Short Answer Type Questions)

3 x 5=15

2. If  ${}^n C_1, {}^n C_2$  and  ${}^n C_3$  are in A.P., then solve the value of  $n$ . (3)

3. Show that the number of ways in which 7 different beads can be arranged to form a (3)

necklace is 360.

OR

Show that  $s$  is a valid conclusion from the premises  $p \rightarrow \neg q, q \vee r, \neg s \rightarrow p, \neg r$ . (3)

4. Explain the purpose of defuzzification. Name at least one method used for defuzzification. (3)

OR

Explain: A complete graph with  $n$  no of vertices consists of  $\frac{n(n-1)}{2}$  number of edges. (3)

5. Illustrate that the sum of the degree of all vertices in a graph is twice the number of edges in the graph (3)

OR

Let  $G$  be a connected planar simple graph with 25 vertices and 60 edges. Calculate the number of regions in  $G$ . (3)

6. Solve the minimum number of edges necessary in a simple planar graph with 15 regions? (3)

OR

Relation  $R$  is defined on integers and  $R = \{a-b \text{ is divisible by } 10 \text{ where } a \text{ and } b \text{ are integers}\}$ . Test whether  $R$  is an equivalence relation. (3)

### Group-C

(Long Answer Type Questions)

5 x 6=30

7. Illustrate that every cyclic group is abelian. (5)
8. Examine if  $\rho$  is an equivalence relation on  $\mathbb{Z}$ , where relation  $\rho$  is defined on a set  $\mathbb{Z}$  by “ $a \rho b$  if and only if  $a-b$  is divisible by 5” for  $a, b \in \mathbb{Z}$  (5)

OR

Define a complete bipartite graph. (5)

9. (5)

If  $R$  is a relation defined by

$$R = \{(x, y) : x - y \text{ is divisible by } 7\}$$

Show that  $R$  is an equivalence relation.

OR

Explain the Konigsberg Bridge Problem and its solution. (5)

10. Develop the following by using mathematical induction: (5)

$$1(1!) + 2(2!) + 3(3!) + \dots + n(n!) = (n + 1)! - 1$$

OR

Determine the truth value of the quantifier  $\exists x, x^2 - 2x + 5 = 0$ ; set of all real numbers being the domain. (5)

11. Explain the truth value of the universal quantifier of the propositional function  $P(x, y)$  stating " $x^2 + y^2 < 12$ " and the domain is  $\{1, 2, 3\}$ . (5)

OR

Let  $S = \{1, \omega, \omega^2\}$  where  $\omega^3 = 1$ . illustrate that  $S$  is an abelian group with respect to multiplication. (5)

12. Without using truth table, Justify that  $\neg(P \vee Q) \vee (\neg P \wedge Q) \vee P$  is a tautology. (5)

OR

Let  $(G, *)$  be a group and let  $a \in G$ . Let  $H$  be the subset of  $G$  defined by  $H = \{x \in G : x * a = a * x\}$ . Conclude that  $H$  is a subgroup of  $G$ . (5)

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