BRAINWARE UNIVERSITY

Term End Examination 2022
Programme – M.Sc.(MATH)-2019/M.Sc.(MATH)-2022
Course Name – Linear Algebra
Course Code - MSCMC101
(Semester I)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) If $\{\alpha, \beta, \gamma\}$ is a basis of a vector space V, then examine $\{\alpha, \beta+\gamma, \gamma\}$.

a) It is a basis of V
b) It is linearly dependent
c) It is linearly independent but not a basis
d) None of these

(ii) Identify the non-linear transformation.

a) $T: R^2 \to R^2 : \mathrm{T(x, y)} = (3x-y, 2x)$
b) $T: R^3 \to R^2 : \mathrm{T(x, y, z)} = (3x+1, \mathrm{y}-\mathrm{z})$
c) $T: \mathrm{R} \to R^2 : \mathrm{T(x)} = (5x, 2x)$
d) $T: \mathrm{R}^3 \to R^2 : \mathrm{T(x, y, z)} = (x, 0, z)$

(iii) Determine a 2x2 orthogonal matrix, whose first row is a multiple of (3, -4).

a) $\begin{bmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}$
b) $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$
c) Both $\begin{bmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}$ & $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$
d) Neither $\begin{bmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}$ nor $\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$

(iv) Evaluate the orthogonally diagonalizable matrix.

a) $\begin{bmatrix} 0 & 2 & 3 \\ 2 & 0 & 4 \\ -3 & 4 & 0 \end{bmatrix}$
b) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 4 \\ 3 & 4 & 3 \end{bmatrix}$
c)
d)

$$\begin{bmatrix} 1 & 2 & 3 \\ -2 & 4 & 4 \\ -3 & -4 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 4 \\ 0 & 0 & 3 \end{bmatrix}$$

(v) Write the sum of the eigen values of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.

a) <p style="text-align: left;"> 5
b) -5
c) 7
d) -7

(vi) If A is an orthogonal Matrix then examine the matrix A.

a) Singular Matrix
b) Non-Singular Matrix
c) Symmetric Matrix
d) Skew-Symmetric matrix

(vii) Evaluate the dimension of the algebra $A(V)$, where $V = M_{3,4}$.

a) 3
b) 4
c) 12
d) 144

(viii) Let $M_{n\times n}$ be the set of all n-square symmetric matrices and the characteristics polynomial of each $A \in M_{n\times n}$ is of the form

$t^n + t^{n-2} + a_{n-3}t^{n-3} + \cdots + a_1 t + a_0$. Then write the dimension of $M_{n\times n}$ over R is

a) $\frac{(n-1)n}{2}$
b) $\frac{(n-2)n}{2}$
c) $\frac{(n-1)(n+2)}{2}$
d) $(n-1)^2/2$

(ix) If 0 is an Eigen value of a matrix A, then identify the false statement.

a) 0 is an Eigen value of A^{-1}
b) 0 is an Eigen value of A^T
c) A has no inverse matrix
d) A can't be orthogonal

(x) Let A is an orthogonal matrix. Evaluate which of the following is not a possible eigen value of A?

a) <p style="text-align: left;"> -1
b) 0
c) 1
d) $\sqrt{-1}$

(xi) If $V = R^3$ be equipped with inner product $(x, y) = x_1y_1 + 2x_2y_2 + 3x_3y_3$, In this inner product space $(V, (.,.))$ identify the pairs of vectors that is orthonormal.

a) $u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

b) $u = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$

c) $u = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ 0 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

d) $u = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, v = \begin{bmatrix} 0 \\ \frac{1}{2} \\ 0 \end{bmatrix}$

(xii) Consider the inner product space of all polynomial of degree less than or equal to 3 and the inner product $f(x).g(x) = \int_{-1}^{1} f(x)g(x)dx$ then determine the value of $x.x^3$.

a) <p style="text-align: left;"> 1/4
b) 1/5
c) 2/5
d) 0

(xiii) For the matrix $A = \begin{pmatrix} 1 & 1 & -1 \\ -2 & 3 & 0 \\ -2 & 1 & 2 \end{pmatrix}$, determine the eigen vector corresponding to the eigen value 3.

a) (0,1,1)
b) (1,2,1)
c) (1,1,1)
d) None of these

(xiv) Select the true statement.

a) Every quadratic form is a bilinear form.
b) If two matrices are congruent, they have the same eigenvalues.
c) Symmetric bilinear forms have symmetric matrix representations.
d) Any symmetric matrix is congruent to a diagonal matrix.

(xv) Examine the invertible matrix.

a) $\begin{pmatrix} 1 & 0 & 2 & 2 \\ 2 & 1 & 3 & 4 \\ 3 & 2 & 4 & 6 \\ 4 & 3 & 5 & 8 \end{pmatrix}$

b) $\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Compute the characteristic polynomial of $\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$. (3)

3. Show that the characteristic polynomial of an operator T does not depend on the choice of a basis. (3)

OR

Let T be a linear operator on a finite-dimensional vector space V. If V has dimension 10. then show that T has at most 10 eigenvalues. (3)

4. Compute the eigenvalues of $\begin{bmatrix} 0 & -i \\ i & 1 \end{bmatrix}$. (3)

OR

Compute the eigenvectors of $\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}$. (3)

5. Test whether A and A^t have the same eigenvectors. (3)

OR

Test whether every complex square matrix A similar to its transpose. (3)

6. Justify the statement, "For any $n \times n$ complex matrix A, the exponential e^A is invertible, and its inverse is e^{-A}." (3)

OR

Justify the statement, "The eigenvalues and the trace of a Hermitian matrix A are real numbers." (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. It is given that 3, 0, 0 are the Eigen values of the matrix $\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$. (5)

Calculate the diagonalizing matrix.

8. State Cauchy-Schwarz inequality, Pythagoras theorem and Parallelogram law. (5)

OR

Define the rank of a matrix A. And also state the rank-nullity theorem. (5)

9. Show that positive definite operator is also self-adjoint. (5)

OR

Let V(R) be a vector space of all 2x 2 matrices over the real field R. Show that W is not a subspace of V where W consists of the set of matrices A for which $A^2 = A$. (5)

10. Determine whether the set of vectors formed by the matrices A, B and C are dependent where $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -5 \\ -4 & 0 \end{bmatrix}$ (5)

OR

(5)

If A is a complex 5x5 matrix with characteristic polynomial $f(x) = (x-2)^3(x+7)^2$ and minimal polynomial is $p(x) = (x-2)^2(x+7)$, then determine the Jordan form for A .

11. Evaluate the eigen vectors of the matrix $\begin{bmatrix} 5 & 1 \\ 4 & 2 \end{bmatrix}$ (5)

OR

Evaluate the rank and signature of $xy + yz + zx$. (5)

12. Evaluate a basis and the dimension of the subspace W of R^3, where $W = \{(x, y, z) \in R^3 \mid x + y + z = 0\}$. (5)

OR

Test whether $W = \{(\alpha_1, \alpha_2, \dots, \alpha_n) \in R^n : \alpha_2 = {\alpha_1}^2\}$ is a subspace of $R^n(R)$. (5)

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