BRAINWARE UNIVERSITY

Term End Examination 2022
Programme – M.Sc.(MATH)-2019/M.Sc.(MATH)-2022
Course Name – Real Analysis
Course Code - MSCMC102
(Semester I)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Assume that $\alpha \uparrow$ on $[a, b]$. If $f \in R(\alpha)$ on $[a, b]$, then select the correct statement.

a) $\left|\int_a^b f(x)d\alpha(x)\right| = \int_a^b |f(x)|d\alpha(x)$

b) $\left|\int_a^b f(x)d\alpha(x)\right| \leq \int_a^b |f(x)|d\alpha(x)$

c) $\int_a^b |f(x)|d\alpha(x) \leq \left|\int_a^b f(x)d\alpha(x)\right|$

d) None of the mentioned

(ii) A function f is defined on $\left(-\frac{1}{3}, \frac{1}{3}\right)$ by $f(x) = 1 + 2.3x + 3.3^2x^2 + \cdots + n.3^{n-1}x^{n-1} + \cdots$ Then write about f.

a) f is continuous on $\left(-\frac{1}{3}, \frac{1}{3}\right)$

b) f is continuous on $\left(-\frac{1}{3}, \frac{1}{3}\right]$

c) f is continuous on $\left[-\frac{1}{3}, \frac{1}{3}\right)$

d) None of the mentioned

(iii) Compute the Cesaro's sum of the series $1 - 1 + 1 - 1 + 1 - 1 + \ldots$

a) 0

b) 0.5

c) 1

d) none of the mentioned

(iv) Compute the Abel's sum of the series $1 - 1 + 1 - 1 + 1 - 1 + \ldots$

a) 0

b) 0.5

c) 1

d) none of the mentioned.

(v) Evaluate $\int_0^{0.25} f$, where $f(x) = 1 + 2.3x + 3.3^2x^2 + \cdots + n.3^{n-1}x^{n-1} + \cdots$

a) 0
b) 1
c) 0.25
d) none of these

(vi) Indicate the range of validity of the series $\sum_{k=0}^{\infty}(2^k + 3^k)x^k$

a) $-\frac{1}{2} < x < \frac{1}{2}$
b) $-\frac{1}{3} < x < \frac{1}{3}$
c) $-\frac{1}{2} \le x < \frac{1}{2}$
d) $-\frac{1}{3} < x \le \frac{1}{3}$

(vii) If $A \in L(R^n, R^m)$, then select the correct statement.

a) $\infty \ge \|A\| > 0$
b) $\infty > \|A\| > 0$
c) $\infty > \|A\| \ge 0$
d) $\infty > \|A\| > -\infty$

(viii) Let Ω be the set of all invertible linear operators on R^n. Then select the correct statement for Ω.

a) Ω closed in $L(R^n)$
b) Ω open in $L(R^n)$.
c) Ω dense in $L(R^n)$.
d) None of the mentioned

(ix) Identify the open subset of **R**?

a) [0,1)
b) (-1, 3]
c) (0, 1)
d) [0,1]

(x) Recognize the correct statement. Every bounded infinite subset of has

a) at most one limit point in **R**
b) at least one limit point in **R**
c) exactly one limit point in **R**
d) None of the mentioned

(xi) Identify the correct statement for the derived set S' of any set S and $A, B \subset \mathbf{R}$.

a) $(A \cap B)' = A' \cap B'$
b) $(A \cap B)' \subset A' \cap B'$
c) $(A \cap B)' \supset A' \cap B'$
d) None of the mentioned

(xii) Let A and B be subsets of **R** such that A be closed and B be compact. Then classify $A \cap B$.

a) $A \cap B$ is compact
b) $A \cap B$ is closed but not compact
c) $A \cap B$ is not closed
d) None of these

(xiii) Evaluate the norm of the operator $A(x, y) = (x, 0)$.

a) <p style="text-align: left;"> 0
b) 0.5
c) 1
d) none of the mentioned

(xiv) Evaluate the norm of the operator $A(x, y) = \left(\frac{x}{2}, \frac{y}{2}\right)$.

a) 0
b) 0.5
c) 1
d) None of the mentioned

(xv) Assume $\alpha \uparrow$ on $[a, b]$. If $f(x) \le g(x)$ on $[a, b]$, then validate the following inequality.

a) $\int_a^b f d\alpha \le \int_a^b g d\alpha$
b) $\int_a^b f d\alpha < \int_a^b g d\alpha$
c) $\int_a^b f d\alpha \ge \int_a^b g d\alpha$
d) $\int_a^b f d\alpha > \int_a^b g d\alpha$

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Show that the set **N** of all positive integers is not bounded above. (3)

OR

Explain Archimedean Property in **R**. (3)

3. Applying definition of compact set show that (0, 1) is not a compact subset of R. (3)

OR

Sketch the prove that a closed and bounded interval is a closed set. (3)

4. Let $G \subset \boldsymbol{R}$ be an open set and $F \subset \boldsymbol{R}$ be a closed set. Explain why $G - F$ is an open set while $F - G$ is a closed set. (3)

OR

Explain why that Riemann integral on [a, b] is a particular type of Riemann-Stieltjes integral on [a, b]. (3)

5. Test whether the set **Z** of all integers is not compact. (3)

OR

Let A and B be subsets of **R** of which A is closed and B is compact. Test whether $A \cap B$ is compact. (3)

6. If the power series $a_0 + a_1x + a_2x^2 + ..$.diverges for$x = x_1$, then validate that the series diverges for all real $x, |x| > |x_1|$. (3)

OR

Let A be a $m \times n$ matrix and if $x \in R^n$, then validate that the derivative of A at $x \in R^n$ is A (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. State Implicit function theorem and Inverse function theorem (5)

OR

State Banach contraction principle and the rank theorem (5)

8. (5)

Show that the function $f(x, y)$ where $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & x^2+y^2 \neq 0 \\ 0 & x = y = 0 \end{cases}$ is not differentiable at the origin.

OR

Discuss about the infinite intersection of open sets. (5)

9. From the equation $2x^2 - yz + xz^2 = 4$ determine $\frac{\partial x}{\partial y}, \frac{\partial x}{\partial z}$ (5)

OR

Using the definition of a compact set, prove that a finite subset of JR is a compact set in **R**. (5)

10. Explain the rectifiable curves. (5)

OR

Calculate the radius of convergence of the power series (5)

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n+1}(x+1)^n.$$

11. If $1 + x + x^2 + \cdots = \frac{1}{x-1}, |x| < 1$ *then deduce* the power series expansion of $log(1-x)$. (5)

OR

Assume the power series expansion of $(1+x)^{-2}$ deduce the power series expansion of $tan^{-1}x$. (5)

12. Evaluate the limit points of the set $\{n : n \in \mathbb{N}\}$. (5)

OR

Test the convergence of the power series $\sum_{n=0}^{\infty} nx^{2n}$. (5)

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