BRAINWARE UNIVERSITY

Term End Examination 2022
Programme – Dip.CSE-2018/Dip.EE-2018/Dip.ECE-2019/Dip.CSE-2019/Dip.ME-2019/Dip.CSE-2020/Dip.CSE-2021/Dip.EE-2021/Dip.ME-2021/Dip.CSE-2022/Dip.ME-2022/Diploma in Robotics & Automation-2022/Dip.EE-2022/Dip.CE-2022
Course Name – Mathematics I/Mathematics-I
Course Code - DMAT010101/DECE104/DCSE104/DME104/DEE104/BS102
(Semester I)

Full Marks : 60   **Time : 2:30 Hours**
[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question)   1 x 15=15

1.   *Choose the correct alternative from the following :*

(i) Observe the value of $\log_8^{\sqrt{8}} =$

a) 1/3   b) 1/4
c) 1/2   d) 1/8

(ii) If $x = a\sec^2\theta$, $y = b\tan^2\theta$, then write the value of $\dfrac{dy}{dx} =$

a) a/b   b) a
c) ab   d) b/a

(iii) If $x = a\cos t$ and $y = b\sin t$, then write the value of $\dfrac{dy}{dx} =$

a) $\dfrac{b}{a}\cot t$ b) $\dfrac{a}{b}\cot t$

c) $-\dfrac{b}{a}\cot t$ d) $\dfrac{b}{a}\tan t$

(iv) If $y = \tan^{-1}\dfrac{\cos x}{1+\sin x}$, then evaluate $\dfrac{dy}{dx} =$

a) 1   b) 1/2
c) -1/2   d) 2

(v) Evaluate $\dfrac{d}{dx}\left\{\cos^{-1}\dfrac{x}{5}\right\} =$

a) $\dfrac{1}{\sqrt{25-x^2}}$ b) $\dfrac{1}{\sqrt{25+x^2}}$

c) $\sqrt{25-x^2}$ d) $-\dfrac{1}{\sqrt{25-x^2}}$

(vi)

If $C = \begin{vmatrix} 1 & 2 & 3 \\ -3 & 0 & -1 \\ 5 & -6 & 7 \end{vmatrix}$, then evaluate the co-factor of the element 2 is

a) 6

b) -16

c) 14

d) 16

(vii)

Determine the value of $\begin{vmatrix} 1 & 1 & 1 \\ 4 & 6 & 9 \\ 5 & 6 & 7 \end{vmatrix}$

a) 0

b) -1

c) 19

d) none

(viii)

Determine the value of $\begin{vmatrix} 1 & -2 & 3 \\ 2 & -1 & 4 \\ -2 & 3 & 1 \end{vmatrix}$

a) 20

b) 13

c) 19

d) none

(ix)

Deetermine the value of $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 4 & 6 & 8 \end{vmatrix}$

a) 0

b) 1

c) 2

d) none of these

(x)

If $A = \begin{bmatrix} 0 & -1 \\ 1 & 3 \\ 2 & 0 \end{bmatrix}$, then evaluate $2A^T =$

a)
$\begin{bmatrix} 0 & 2 & 4 \\ -2 & 6 & 0 \end{bmatrix}$

b)
$\begin{bmatrix} 0 & -2 \\ 2 & 6 \\ 4 & 0 \end{bmatrix}$

c)
$\begin{bmatrix} 0 & 1 & 2 \\ -2 & 6 & 0 \end{bmatrix}$

d)
$\begin{bmatrix} 0 & -1 \\ 1 & 3 \\ 4 & 0 \end{bmatrix}$

(xi)

If the co-ordinate of two points A and B are (1,0,5) and (-1,5,2) respectively then

Identify the value of $\vec{AB} =$

a) $5i + 7k$

b) $-2i + 5j - k$

c) $3i + 2j - k$

d) $-2i + 5j - 3k$

(xii) Select the correct value for logarithm of 1728 to the base $2\sqrt{3}$

a) 3

b) 6

c) 9

d) none

(xiii) Identify the vector in the direction of the vector $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9

a)
$$\hat{i} - 2\hat{j} + 2\hat{k}$$

b)
$$\frac{1}{3}\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$$

c)
$$3\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$$

d)
$$9\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$$

(xiv) If $\sin x = \dfrac{3}{4}$ then identify the value of $\cos x$

a)
$$\frac{2}{3}$$

b)
$$\frac{\sqrt{3}}{2}$$

c)
$$\frac{\sqrt{7}}{4}$$

d)
$$\frac{1}{2}$$

(xv) If $\log_x^{81} = 4$ then tell the value of x=

a) 1

b) 2

c) 3

d) none

Group-B
(Short Answer Type Questions)   3 x 5=15

2. Show that $(1-\omega)(1-\omega^4)(1-\omega^2)(1-\omega^5) = 9$.   (3)

OR

Rewrite $\left(\dfrac{1+i}{1-i}\right)^3$ in the form A + iB where A and B are real numbers.   (3)

3. Justify that the points $\vec{A} = -2\,\vec{i} + 3\,\vec{j} + 5\,\vec{k}$, $\vec{B} = \vec{i} + 2\,\vec{j} + 3\,\vec{k}$ and $\vec{C} = 7\vec{i} - \vec{k}$ are collinear.   (3)

OR

Write the area of the triangle with vertices A(1,1,2), B(2,3,5) and C(1,5,5).   (3)

4. Establish that each of the given three vectors is a unit vector :   (3)
$$\frac{1}{7}\left(2\hat{i} + 6\hat{j} + 6\hat{k}\right), \frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right), \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right).$$

OR

Using vectors , write the value of k such the points (k,-10,3),(1,-1,3) and (3,5,3) ar collinear.   (3)

5. Evaluate $\dfrac{dy}{dx}$ when $y = \sqrt{\dfrac{\sec x - \tan x}{\sec x + \tan x}}$. (3)

<div style="text-align:center">OR</div>

Evaluate the limit : $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{\sqrt{5x - 1} - \sqrt{3x + 1}}$. (3)

6. Write the value of $\dfrac{dy}{dx}$ if $y = \tan^{-1}\sqrt{\dfrac{1 - x}{1 + x}}$ (3)

<div style="text-align:center">OR</div>

Let $f(x) = \dfrac{x^2 - 1}{x^3 - 1}, x \neq 1$. Write the value of $f(x)$ at x=1 such that $f(x)$ be continuous at x=1 ? (3)

<div style="text-align:center">**Group-C**</div>
<div style="text-align:center">(Long Answer Type Questions)   5 x 6=30</div>

7. If vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $\vec{a} + \vec{b} + \vec{c} = 0$, $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 5$ and $\left|\vec{c}\right| = 7$ tell the angle between $\vec{a}, \vec{b}$. (5)

<div style="text-align:center">OR</div>

Visualize the equation of straight line which passes through the point (1 , -2) and cuts off equal intercepts from axes? (5)

8. If $\dfrac{\log x}{ry - qz} = \dfrac{\log y}{pz - rx} = \dfrac{\log z}{qx - py}$ show that $x^p y^q z^r = 1$. (5)

<div style="text-align:center">OR</div>

Show that $\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd\left(1 + \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c} + \dfrac{1}{d}\right)$ (5)

9. (5)

If $\sin^4\theta + \sin^2\theta = 1$ establish that $\tan^2\theta - \tan^4\theta + 1 = 0$.

(5)

Establish that $\sqrt{\dfrac{\text{cosecx} + 1}{\text{cosecx} - 1}} = \dfrac{\cos x}{1 - \sin x}$.

10.

$$f(x) = \begin{array}{ll} x & x \geq 0 \\ -x & x < 0 \end{array}$$

Let

(5)

Analyze that f(x) is continuous at x=0 but not derivable at that point.

(5)

Find , from definition , evaluate the derivative of $\log 2x$

11.

If $f(x) = \log\dfrac{1+x}{1-x}$ deduce that $f\left(\dfrac{2x}{1+x^2}\right) = 2f(x)$.

(5)

(5)

Evaluate $\dfrac{dy}{dx}$ if $y = x^2\sqrt{\dfrac{x^2 - x + 1}{x^2 + x + 1}}$

12. Evaluate the value of k , so that the function f defined below is continuous at (5)

x=0 , where $f(x) = \begin{cases} \dfrac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & -1 \leq x < 0 \\ \dfrac{2x+1}{x-1}, & 0 \leq x < 1 \end{cases}$

(5)

Evaluate the value of K for which the function $f(x) = \begin{cases} Kx^2 + 2x, & x < 2 \\ Kx^3, & x \geq 2 \end{cases}$ is

continuous at x=2 .

****