BRAINWARE UNIVERSITY

Term End Examination 2022
Programme – M.Tech.(CSE)-AIML-2022
Course Name – Mathematics -I
Course Code - BSC-MMM101
(Semester I)

Full Marks : 60 **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Solve the standard deviation of a Poisson distribution with mean 4.

a) 4
b) 3
c) 2
d) 16

(ii) Solve the mean of a binomial distribution with n=16 and p=0.5

a) 8
b) 4
c) 2
d) 16

(iii) Select the correct option: The arrival and departure of quests in a hotel in queueing systems can be stated as

a) The pure birth process.
b) The pure death process.
c) The birth death process.
d) None of these.

(iv) Choose the correct option: In certain situation the customer arrives from one gate and departure takes from another gate. This situation of incoming and outgoing can be stated as

a) The pure death process.
b) The pure birth process.
c) The birth-death process.
d) None of these.

(v) Suppose that vehicle speeds at an interstate location have a normal distribution with a mean equal to 70 mph and standard deviation equal to 8 mph. Evaluate the z-score for a speed of 78 mph.

a) 1
b) -1
c) 2
d) 0

(vi) Choose the correct option: There are N inventories in the system, one by one all the inventories are consumed with replacing the inventories. This process is stated as

a) The pure birth process.
b) The pure death process.
c) The birth death process.
d) None of these

(vii) A problem is given to 2 students P, Q. If the probability of solving the problem individually is 1/2, 1/3 respectively, evaluate the probability that the problem is solved.

a) 1/3
b) 2/3
c) 0
d) 1

(viii) The probability of success in a Bernoulli trial is 0.3. Calculate the variance.

a) 0.09
b) 0.21
c) 0.9
d) 0.021

(ix) Identify the right option: The middle value of an ordered array of numbers is the

a) Mode
b) Mean
c) Median
d) Mid-point

(x) Select the right option: The steady-state probability vector π of a discrete Markov chain with transition probability matrix P satisfies the matrix equation

a) $P\pi = 0$

b) $(1-P)\pi = 0$

c) $P\pi = \pi$

d) $P^t\pi = 0$

(xi) Consider the situation, when no server is working, then calculate the number of customers in the system

a) Equal to number of customers in queue.

b) More than number of customers in queue.

c) Less than number of customers in queue.

d) None of these.

(xii) Select the correct option: In descriptive statistics, we study

a) The description of decision making process

b) The methods for organizing, displaying, and describing data

c) How to describe the probability distribution

d) None of these

(xiii) Select the correct option: The condition for independence of two events A and B is

a) $P(A\cap B) = P(A)P(B)$

b) $P(A+B) = P(A)P(B)$

c) $P(A-B) = P(A)P(B)$

d) $P(A\cap B) = P(A)P(B/A)$

(xiv) Select the correct option: A coin is tossed .The events {H}, {T} are

a) mutually exclusive

b) independent events

c) dependent events

d) None of these

(xv) Identify the right option: The first order Markov chain is generally used, when

a) transition probabilities are fairly stable

b) change in transition probabilities is random

c) no sufficient data are available

d) all of the above will hold

Group-B

(Short Answer Type Questions) 3 x 5=15

2. If A and B are events with $P(A) = \frac{3}{8}, P(B) = \frac{5}{8}$ and $P(A\cup B) = \frac{3}{4}$, compute $P(A/B)$. (3)

3. Explain recurrent and transient states in a Markov chain. (3)

OR

Explain communicative states in a Markov chain. (3)

4. The weight of students in a college is normally distributed with m=40kg, σ=5kg. Compute the percentage of the students that have weight (3)

a) In the range of 38 kg and 52 kg

b) Greater than 50 kg

c) Greater than 40 kg.

Given $\Phi(2) = 0.9772$, $\Phi(0.4) = 0.6554$, $\Phi(2.4) = 0.9918$.

OR

Let X be a random variable with following probability distribution: (3)

x:	-3	6	9
P(X=x):	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Compute $E[X]$ and $E[X^2]$.

5. In a reliability test there is a 42% probability that a computer chip survives more than 500 temperature cycles. If a computer chip does not survive more than 500 temperature cycles, then there is a 73% probability that it was manufactured by company A. Evaluate the probability that a computer chip is not manufactured by company A and does not survive more than 500 temperature cycles? (3)

OR

Show that, (3)

$$f(x)=x, \quad 0\le x<1$$
$$=k-x, \quad 1\le x\le 2$$
$$=0 \quad elsewhere$$

is a p.d.f of a random variable X then find the value of k. Evaluate the probability that the random variable lies between $\frac{1}{2}$ and $\frac{3}{2}$.

6. If X be a continuous random variable then justify that- $\lim_{x\to\infty} F(x)=1$ (3)

OR

Justify that the mean and variance for a Poisson distribution with parameter 'm' will be equal to each other's. (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Describe the axiomatic definition of probability. (5)

8. A random variable X has the following probability function values: (5)

x	0	1	2	3	4	5	6	7
P(X=x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) Evaluate the value(s) of k.

(ii) Evaluate P(X<6).

9. Let A and B be two events. Suppose the probability that neither A or B occurs is 2/3. Estimate is the probability that one or both occur? (5)

OR

Let C and D be two events with P(C) = 0.25, P(D) = 0.45, and P(C ∩ D) = 0.1. (5)

Estimate is the value of $P(C^C \cap D)$?

10. The probability density function of a continuous distribution is given by (5)

$f(x) = \frac{3}{4}x(2-x), 0 < x < 2.$ Compute mean

OR

You roll one red die and one green die. Define the random variables X and Y as follows: (5)

X = The number showing on the red die

Y = The number of dice that show the number six

For example, if the red and green dice show the numbers 6 and 4, then X = 6 and Y = 1. Write down a table showing the joint probability mass function for X and Y, compute the marginal distribution for Y , and compute E(Y).

11. Evaluate the mean, variance and standard deviation of a Binomial distribution with parameter n and p. (5)

OR

Evaluate the mean and variance for a normal distribution. (5)

12. If A and B are two independent events, then justify that (5)

i) A^c and B^c are independent.

ii) A^c and B are also independent.

OR

Let X and Y be two continuous random variables with joint pdf (5)

$f(x, y) = cx^2y(1 + y)$ for $0 \le x \le 3$ and $0 \le y \le 3$, and f(x, y) = 0 otherwise

(a) Evaluate the value of c.

(b) Evaluate the probability $P(1 \le X \le 2, 0 \le Y \le 1)$.

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