BRAINWARE UNIVERSITY

Term End Examination 2022

Programme – B.Tech.(CSE)-2018/B.Tech.(ECE)-2018/B.Tech.(ECE)-2019/B.Tech.(CSE)-2019/B.Tech.(CSE)-2020/B.Tech.(ECE)-2020/B.Tech.(RA)-2022

Course Name – Calculus/Calculus & Linear Algebra

Course Code - BMAT010101/BSC(ECE)101/BSC(CSE)101/BSCR102

(Semester I)

Full Marks : 60 **Time : 3:0 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question) 1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Let $\sum_{n=1}^{\infty} a_n$ be an infinite series of positive terms. If $\lim_{n\to\infty} a_n = 5$, then test the series is

a) convergent b) Divergent

c) divergent and convergent d) None of these

(ii) If f(x) satisfy all the conditions of Rolle's theorem in $[a,b]$, then choose, $f'(x)$ becomes zero

a) only at one point in (a,b) b) at two points in (a,b)

c) at least one point in (a,b) d) None of these

(iii)

Write the value of $\beta\left(\frac{1}{2},\frac{1}{2}\right)$ is

a) π b) $\sqrt{\pi}$

c) $\frac{\sqrt{\pi}}{2}$ d) $\frac{\pi}{2}$

(iv)

Write the sum of Fourier series of the function $f(x) = x + x^2, -\pi < x < \pi$ at the point $x = \pi$ as:

a) $\pi + \pi^2$ b) π

c) π^2 d) 0

(v) For $k > 0, n > 0$, Calculate $\int_1^\infty \frac{(\log y)^{n-1}}{y^{k+1}} dy$

a) $\frac{\Gamma(n)}{k^n}$ b) $\frac{\Gamma(k)}{k^n}$

c) $\frac{\Gamma(k)}{n^n}$ d) None of these

(vi) If $u(x,y) = \frac{x^3 + y^3}{\sqrt{x+y}}$ then compile that $x\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} =$

a) 1/2 b) 5/2

c) $\frac{5}{2}u(x,y)$ d) $\frac{1}{2}u(x,y)$

(vii) Evaluate $\lim_{x \to 0+}\left(\frac{1}{x} - \frac{1}{\sin x}\right) =$

a) 1 b) 0

c) 2 d) None of these

(viii) Select the value of the triple integral $\int_0^1 \int_5^8 \int_0^9 dx\,dy\,dz$ is

a) 25 b) 27

c) 1 d) 3

(ix) Select the Lagrange's form of remainder in Taylor's theorem is

a) $\frac{h^n(1-\theta)^{(n-1)}}{(n-1)!} f^n(a+\theta h)$ b) $\frac{h^n(1-\theta)^{(n-p)}}{p(n-1)!} f^n(a+\theta h)$

c) $\frac{h^n}{n!} f^n(a+\theta h)$ d) None of these

(x) If θ be the angle between the vectors $\vec{a} = 6\hat{i} + 2\hat{j} + 3\hat{k}$ & $\vec{b} = 2\hat{i} - 9\hat{j} + 6\hat{k}$, then select from the following

a) $\theta = \cos^{-1}\left(\frac{12}{77}\right)$ b) $\theta = \tan^{-1}\left(\frac{12}{77}\right)$

c) $\theta = cos^{-1}\left(\frac{77}{12}\right)$ d) none of these

(xi) Test, if $\vec{a}.(\vec{b} \times \vec{c}) = 0$, then the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are

a) independent b) coplanar

c) collinear d) none of these

(xii) Select from the following: If $\phi(x,y) = x^2 - 2xy + y^3$, then $\vec{\nabla}\phi$ at $(2,3)$ is

a) $-2\hat{i} + 2\hat{j}$ b) $-2\hat{i} - 2\hat{j}$

c) $2\hat{i} - 2\hat{j}$ d) $2\hat{i} + 2\hat{j}$

(xiii)

Test the improper integral $\int_0^\infty \frac{\cos mx}{x^2+a^2}dx,\ m>0,\ a>0$ is

a) Convergent
b) Divergent
c) 1
d) None of these

(xiv) Show the characteristic points of the circles $(x-\alpha)^2+y^2=\alpha^2$ are

a) $(\alpha,\pm a)$
b) $(\pm\alpha,a)$
c) $(\pm\alpha,-a)$
d) None of these

(xv) Test the sequence $\left\{\frac{1}{3^n}\right\}$ is

a) Monotonic increasing
b) Monotonic decreasing
c) Oscillatory
d) None of these

Group-B

(Short Answer Type Questions) 3 x 5=15

2. Deduce the extrema of the following function: (3)

$$f(x,y)=x^3+3xy^2-3y^2-3x^2+4$$

3. If $z=\tan(y+ax)-(y-ax)^{\frac{3}{2}}$, show that $\frac{\partial^2 z}{\partial x^2}=a^2\frac{\partial^2 z}{\partial y^2}$. (3)

4. Change the order of integration and hence evaluate $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}dydx$. (3)

5. In the mean value theorem $f(h)=f(0)+hf'(\theta h)$, $0<\theta<1$, show that the limiting value of θ as $h\to 0$ is $\frac{1}{2}$ if $f(x)=\cos x$ (3)

OR

Use mean value theorem to show $0<\frac{1}{x}\log\frac{e^x-1}{x}<1$, for $x>0$. (3)

6. Prove that if a function $f:[a,b]\to\mathbb{R}$ be continuous on $[a,b]$, differentiable on (a,b) and if $f'(x)=0$ for all $x\in(a,b)$ then f is constant on $[a,b]$. (3)

OR

Use Stoke's theorem to prove that $div\, curl\, \bar{F} = 0$ (3)

Group-C

(Long Answer Type Questions) 5 x 6=30

7. Find the Fourier series expansion for the function $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$. Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$ (5)

8. Show that $2^{2m-1}\Gamma(m)\Gamma(m+\frac{1}{2}) = \sqrt{\pi}\,\Gamma(2m)$ (5)

9. If $u = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$, then show that (5)

   (i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = xf\left(\frac{y}{x}\right)$

   (ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$.

10. Show that the series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots\ldots$ converges for $p > 1$ and diverges for $p \le 1$. (5)

11. a. Evaluate $\iiint (x+y+z+1)^4 dxdy\, dz$ over the region bounded by $x \ge 0,\ y \ge 0,\ z \ge 0,\ x+y+z \le 1$. (5)

    b. Show that $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \frac{\pi}{\sqrt{2}}$.

OR

Apply Gauss's Divergence Theorem to evaluate (5)

$\iint_S [(x^3 - yz)\hat{\imath} - 2x^2y\hat{\jmath} + 2\hat{k}] \cdot \hat{n} dS$

where S denotes the surface of the cube bounded by the planes $x = 0, x = a, y = 0, y = a, z = 0, z = a$ and $\hat{n}$ is the unit outward normal to S.

12. (5)

A fluid motion is given by

$\vec{v} = (ysinz - sinx)\hat{\imath} + (xsinz + 2yz)\hat{\jmath} + (xycosz + y^2)\hat{k}$.

Is the motion irrotational? Test. If so, find the velocity potential.

OR

Show that $\overrightarrow{f} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational. Find a scalar function φ such that $\overrightarrow{f} = \vec{\nabla}\varphi$. (5)

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