



BRAINWARE UNIVERSITY

Term End Examination 2022

Programme – Dip.CSE-2018/Dip.EE-2018/Dip.ECE-2019/Dip.CSE-2019/Dip.ME-2019/Dip.CSE-2020/Dip.CSE-2021/Dip.EE-2021/Dip.ME-2021/Dip.CSE-2022/Dip.ME-2022/Diploma in Robotics & Automation-2022/Dip.EE-2022/Dip.CE-2022

Course Name – Mathematics I/Mathematics-I

Course Code - DMAT010101/DECE104/DCSE104/DME104/DEE104/BS102

(Semester I)

Full Marks : 70

Time : 3:0 Hours

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question)

1 x 15=15

1. Choose the correct alternative from the following :

(i) Observe the value of $\log_8 \sqrt[3]{8} =$

- a) 1/3
- b) 1/4
- c) 1/2
- d) 1/8

(ii) If $x = a \sec^2 \theta$, $y = b \tan^2 \theta$, then write the value of $\frac{dy}{dx} =$

- a) a/b
- b) a
- c) ab
- d) b/a

(iii) If $x = a \cos t$ and $y = b \sin t$, then write the value of $\frac{dy}{dx} =$

- a) $\frac{b}{a} \cot t$
- b) $\frac{a}{b} \cot t$
- c) $-\frac{b}{a} \cot t$
- d) $\frac{b}{a} \tan t$

(iv) If $y = \tan^{-1} \frac{\cos x}{1 + \sin x}$, then evaluate $\frac{dy}{dx} =$

- a) 1
- b) 1/2
- c) -1/2
- d) 2

(v) Evaluate $\frac{d}{dx} \left\{ \cos^{-1} \frac{x}{5} \right\} =$

- a) $\frac{1}{\sqrt{25-x^2}}$
- b) $\frac{1}{\sqrt{25+x^2}}$
- c) $\frac{1}{\sqrt{25-x^2}}$
- d) $-\frac{1}{\sqrt{25-x^2}}$

(vi)

If $C = \begin{vmatrix} 1 & 2 & 3 \\ -3 & 0 & -1 \\ 5 & -6 & 7 \end{vmatrix}$, then evaluate the co-factor of the element 2 is

- a) 6
c) 14
- b) -16
d) 16

(vii) Determine the value of $\begin{vmatrix} 1 & 1 & 1 \\ 4 & 6 & 9 \\ 5 & 6 & 7 \end{vmatrix}$

- a) 0
c) 19
- b) -1
d) none

(viii) Determine the value of $\begin{vmatrix} 1 & -2 & 3 \\ 2 & -1 & 4 \\ -2 & 3 & 1 \end{vmatrix}$

- a) 20
c) 19
- b) 13
d) none

(ix) Determine the value of $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 4 & 6 & 8 \end{vmatrix}$

- a) 0
c) 2
- b) 1
d) none of these

(x) If $A = \begin{bmatrix} 0 & -1 \\ 1 & 3 \\ 2 & 0 \end{bmatrix}$, then evaluate $2A^T =$

- a) $\begin{bmatrix} 0 & 2 & 4 \\ -2 & 6 & 0 \end{bmatrix}$
- b) $\begin{bmatrix} 0 & -2 \\ 2 & 6 \\ 4 & 0 \end{bmatrix}$
- c) $\begin{bmatrix} 0 & 1 & 2 \\ -2 & 6 & 0 \end{bmatrix}$
- d) $\begin{bmatrix} 0 & -1 \\ 1 & 3 \\ 4 & 0 \end{bmatrix}$

(xi) If the co-ordinate of two points A and B are (1,0,5) and (-1,5,2) respectively then Identify the value of $\vec{AB} =$

- a) $5i + 7k$
- b) $-2i + 5j - k$
- c) $3i + 2j - k$
- d) $-2i + 5j - 3k$

(xii) Select the correct value for logarithm of 1728 to the base $2\sqrt{3}$

- a) 3
c) 9
- b) 6
d) none
- (xiii) Identify the vector in the direction of the vector $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9

- a) $\hat{i} - 2\hat{j} + 2\hat{k}$
- b) $\frac{1}{3}(\hat{i} - 2\hat{j} + 2\hat{k})$
- c) $3(\hat{i} - 2\hat{j} + 2\hat{k})$
- d) $9(\hat{i} - 2\hat{j} + 2\hat{k})$

- (xiv) If $\sin x = \frac{3}{4}$ then identify the value of $\cos x$

- a) $\frac{2}{3}$
- b) $\frac{\sqrt{3}}{2}$
- c) $\frac{\sqrt{7}}{4}$
- d) $\frac{1}{2}$

- (xv) If $\log_x^{81} = 4$ then tell the value of $x =$

- a) 1
c) 3
- b) 2
d) none

Group-B

(Short Answer Type Questions)

3 x 5=15

2. Evaluate $\frac{dy}{dx}$ when $y = \sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$. (3)

OR

- Evaluate the limit : $\lim_{x \rightarrow 1} \frac{x^2 - 1}{\sqrt{5x - 1} - \sqrt{3x + 1}}$. (3)

3. Show that $(1 - \omega)(1 - \omega^4)(1 - \omega^2)(1 - \omega^5) = 9$. (3)

OR

- Rewrite $\left(\frac{1+i}{1-i}\right)^3$ in the form $A + iB$ where A and B are real numbers. (3)

4. Justify that the points $\vec{A} = -2\vec{i} + 3\vec{j} + 5\vec{k}$, $\vec{B} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{C} = 7\vec{i} - \vec{k}$ are collinear. (3)

OR

- Write the area of the triangle with vertices $A(1,1,2)$, $B(2,3,5)$ and $C(1,5,5)$. (3)

5. Establish that each of the given three vectors is a unit vector : (3)

$$\frac{1}{7}\left(\hat{i}+6\hat{j}+6\hat{k}\right), \frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right), \frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right).$$

OR

Using vectors, write the value of k such the points (k,-10,3),(1,-1,3) and (3,5,3) are collinear. (3)

6. Write the value of $\frac{dy}{dx}$ if $y = \tan^{-1} \sqrt{\frac{1-x}{1+x}}$ (3)

OR

Let $f(x) = \frac{x^2-1}{x^3-1}$, $x \neq 1$. Write the value of $f(x)$ at $x=1$ such that $f(x)$ be continuous at $x=1$? (3)

Group-C

(Long Answer Type Questions)

5 x 8=40

7. Show that the matrix $\begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix} = P + Q$ where P is a symmetric and Q is a skew-symmetric matrix. (5)

8. Establish that $\sin(A+B).\sin(A-B)+\sin(B+C).\sin(B-C)+\sin(C+A).\sin(C-A)=0$ (5)

9. Evaluate the value of k, so that the function f defined below is continuous at $x=0$, where $f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & -1 \leq x < 0 \\ \frac{2x+1}{x-1}, & 0 \leq x < 1 \end{cases}$ (5)

10. Evaluate the value of K for which the function $f(x) = \begin{cases} Kx^2 + 2x, & x < 2 \\ Kx^3, & x \geq 2 \end{cases}$ is continuous at $x=2$. (5)

11. (5)

If $\frac{\log x}{ry-qz} = \frac{\log y}{pz-rx} = \frac{\log z}{qx-py}$ show that $x^p y^q z^r = 1$.

OR

Show that
$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} \right)$$
 (5)

12. If $\sin^4 \theta + \cos^4 \theta = 1$ establish that $\tan^2 \theta - \sec^4 \theta + 1 = 0$. (5)

OR

Establish that
$$\sqrt{\frac{\operatorname{cosec} x + 1}{\operatorname{cosec} x - 1}} = \frac{\cos x}{1 - \sin x}$$
 (5)

13. $f(x) = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$ (5)
 Let

Analyze that $f(x)$ is continuous at $x=0$ but not derivable at that point.

OR

Find, from definition, evaluate the derivative of $\log 2x$ (5)

14. If $f(x) = \log \frac{1+x}{1-x}$ deduce that $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$ (5)

OR

Evaluate $\frac{dy}{dx}$ if $y = x^2 \sqrt{\frac{x^2 - x + 1}{x^2 + x + 1}}$ (5)
