Term End Examination 2022
Programme – B.Tech.(ECE)-2019/B.Tech.(ECE)-2020
Course Name – Probability Theory and Stochastic Process
Course Code - PCC-EC503
(Semester V)

Full Marks : 60   **Time : 2:30 Hours**
[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A
(Multiple Choice Type Question)   1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Let $\{N(t):t \geq 0\}$ be a time homogeneous Poisson Process with rate 2. Then evaluate
$P(N(1) = 5, N(3) = 8, N(6) = 10)$

a) $\dfrac{e^{-12} \times 2^{11} \times 3}{5!}$

b) $\dfrac{e^{-9} \times 2^8 \times 3}{8!}$

c) $\dfrac{e^{-3} \times 2^8 \times 3}{8!}$

d) None of these

(ii) Choose the right option: For a discrete-time Markov chain, let P be the matrix of transition probabilities. The sum of each...
a) column is 1
b) row is 1
c) column is 0
d) row is 0

(iii) Select the right option: The steady-state probability vector π of a discrete Markov chain with transition probability matrix P satisfies the matrix equation

a) $P\pi = 0$

b) $(1-P)\pi = 0$

c) $P\pi = \pi$

d) $P^t \pi = 0$

(iv) Select the correct option: The condition for independence of two events A and B is

a) $P(A \cap B) = P(A)P(B)$

b) $P(A+B) = P(A)P(B)$

c) $P(A-B) = P(A)P(B)$

d) $P(A \cap B) = P(A)P(B/A)$

(v) Select the correct option: The variance of a random variable x is

a) $\{E(x)\}^2$

b) $E(x^2)$

c) $E(x^2) - \{E(x)\}^2$

d) None of these

(vi) Identify the right option: The middle value of an ordered array of numbers is the
a) Mode
b) Mean
c) Median
d) Mid-point

(vii) Identify the right option: Number of times each value appears is called value's
a) range
b) frequence
c) mode
d) standard deviation

(viii) Select the correct option: The sum of frequencies for all classes will always equal
a) 1
b) the number of elements in the dataset
c) the number of classes
d) a number between 0 to 1

(ix) Solve the standard deviation of a Poisson distribution with mean 4.
a) 4
b) 3
c) 2
d) 16

(x) Solve the variance of a Uniform distribution(0,6)
a) 6
b) 3
c) 2
d) 4

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(xi) A problem is given to 2 students P, Q. If the probability of solving the problem individually is 1/2, 1/3 respectively, evaluate the probability that the problem is solved.

    a) 1/3   b) 2/3
    c) 0   d) 1

(xii) Suppose that vehicle speeds at an interstate location have a normal distribution with a mean equal to 70 mph and standard deviation equal to 8 mph. Evaluate the z-score for a speed of 78 mph.

    a) 1   b) -1
    c) 2   d) 0

(xiii) Choose the correct option: In Markov analysis state probabilities must

    a) sum to 1   b) be less than 1
    c) be greater than 1   d) none of these

(xiv) Identify the right option: For an ergodic process

    a) mean is necessarily zero   b) mean square value is infinity
    c) all time averages are zero   d) mean square value is independent if sine

(xv) Choose the right option: In the long run, the state probabilities become 0 & 1

    a) in no cases   b) in some cases
    c) in all cases   d) can not say

Group-B
(Short Answer Type Questions)
3 x 5=15

2. Explain random process with relevant example. (3)

OR

Explain periodic and aperiodic states in a Markov chain. (3)

3. Explain the concept of 'Stochastic Process' briefly. (3)

OR

Patients arrive in a surgery according to a homogeneous Poisson Process with intensity 6 patients an hour. The doctor starts to examine the patients only when the third patient arrives. Compute the expected time from the opening of the surgery until the first patient starts to be examined. (3)

4. Explain absorbing Markov chain. Give an example. (3)

OR

If A and B are events with $P(A) = \dfrac{3}{8}, P(B) = \dfrac{5}{8}$ and $P(A \cup B) = \dfrac{3}{4}$, compute $P(A / B)$. (3)

5. In a reliability test there is a 42% probability that a computer chip survives more than 500 temperature cycles. If a computer chip does not survive more than 500 temperature cycles, then there is a 73% probability that it was manufactured by company A. Evaluate the probability that a computer chip is not manufactured by company A and does not survive more than 500 temperature cycles? (3)

OR

In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total output. Of their output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product. Evaluate the probability that it is defective? (3)

6. If X be a continuous random variable then justify that- $\lim\limits_{x \to \infty} F(x)=1$ (3)

OR

Justify that the matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \dfrac{1}{2} & \dfrac{1}{2} & 0 \end{bmatrix}$ is the TPM of an irreducible Markov chain. (3)

Group-C
(Long Answer Type Questions)
5 x 6=30

7. A random variable X has the following probability function values: (5)

x	0	1	2	3	4	5	6	7
P(X=x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) Evaluate the value(s) of k.

(ii) Evaluate P(X<6).

8. The probability density function of a continuous distribution is given by

$$f(x) = \frac{3}{4}x(2-x), 0 < x < 2.$$

Compute mean

(5)

9. Let X and Y be two continuous random variables with joint pdf

$f(x, y) = cx^2y(1-y)$ for $0 \leq x \leq 3$ and $0 \leq y \leq 3$, and $f(x, y) = 0$ otherwise

(a) Evaluate the value of c.

(b) Evaluate the probability $P(1 \leq X \leq 2, 0 \leq Y \leq 1)$.
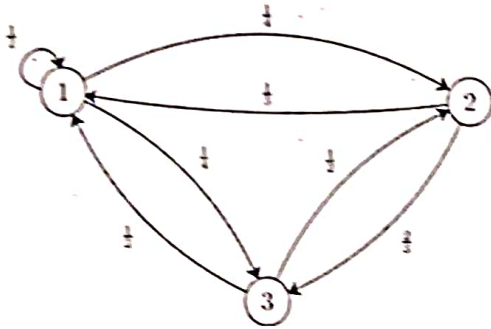
(5)

10. Describe Pure birth process with an example.

(5)

OR

Describe the concept of random walk with an example.

(5)

11. Consider the Markov chain shown in Figure

(5)

a) Discuss if this chain is reducible.

b) Discuss if this chain is aperiodic.

OR

Consider the Markov chain shown in Figure

(5)
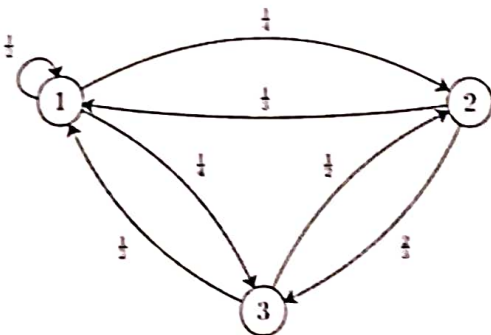
A) Express the stationary distribution for this chain

B) Is the stationary distribution a limiting distribution for the chain?

12. Evaluate the mean, variance and standard deviation of a Binomial distribution with parameter n and p.

(5)

OR

If the random variable X takes the values 1, 2, 3 and 4 such that $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$, evaluate the probability distribution of X.

(5)

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