BRAINWARE UNIVERSITY

Term End Examination 2023
Programme – M.Sc.(MATH)-2022
Course Name – Abstract Algebra
Course Code - MSCMC201
(Semester II)

Full Marks : 60   **Time : 2:30 Hours**

[The figure in the margin indicates full marks. Candidates are required to give their answers in their own words as far as practicable.]

Group-A

(Multiple Choice Type Question)   1 x 15=15

1. *Choose the correct alternative from the following :*

(i) Identify the number of elements of order 2 in a cyclic group of even order is

a) 1   b) 2
c) 4   d) 6

(ii) Evaluate the number of abelian groups upto isomorphism of order 10^5

a) 2   b) 5
c) 7   d) 49

(iii) Identify the correct option. If and are two subgroups of , then

a) HK is a subgroup but $H \cap K$ may not be a subgroup
b) $H \cap K$ is a subgroup but HK may not be a subgroup
c) Both HK and $H \cap K$ are subgroups
d) neither HK nor $H \cap K$ are subgroups

(iv) If a cyclic group has only three subgroups which are {e}, a subgroup of order 7 and the group itself. Then, identify the order of the group is

a) 7   b) 14
c) 49   d) None of these

(v) Identify the correct option. In $(\mathbb{Z}, +)$, the order of all inverse elements is

a) zero   b) one
c) two   d) infinite

(vi) Identify the correct option. Which of the following is not true?

a) Every group of prime order is cyclic

b) A cyclic group is of prime order

c) Every group of order less than 6 is commutative

d) If be a cyclic group of prime order, then every non-identity elements of is a generator of the group.

(vii) Write the correct option. Extension of $\mathbb{Q}$

a) $\mathbb{N}$

b) $\mathbb{Z}$

c) $\mathbb{Q}$

d) $\mathbb{R}$

(viii) If is a group of even order, then select the correct option from below for it has element of order two

a) at least one

b) at least two

c) exactly two

d) at most two

(ix) Select the correct option. $(m\mathbb{Z}, +)$ is a commutative group, then

a) m is prime

b) m is rational

c) m is real

d) m is natural number

(x) The number of generators of the cyclic group $(\mathbb{Z}_{10}, +)$ is

a) 3

b) 4

c) 2

d) 5

(xi) Let be a group of order 1225, then select the correct option for the group from below

a) abelian but not cyclic group

b) cyclic group

c) simple group

d) None of these

(xii) Write the correct option. Extension of $\mathbb{Q}$

a) $\mathbb{N}$

b) $\mathbb{Z}$

c) $\mathbb{Q}$

d) $\mathbb{C}$

(xiii) Write the cardinality of the centre of $\mathbb{Z}_{12}$

a) 1

b) 2

c) 3

d) 12

(xiv) If $c = (1,2,3,4)$, then calculate c^2

a) $(1,3)(2,4)$

b) $(1,3)$

c) $(2,4)$

d) $(2,3)(3,1)$

(xv) If G is a group of even order, for all $a \neq e$ if $a^2 = e$, then select the correct option for the group G

a) abelian group

b) subgroup

c) normal group

d) None of these

Group-B

(Short Answer Type Questions)   3 x 5=15

2. Describe permutation group. State Cayley's theorem.   (3)

3. Test that every proper subgroup of a group of order 6 is cyclic.   (3)

4. Define nilpotent element of a ring. Show that a ring containing no divisor of zero contains no non-zero nilpotent element.   (3)

5. Examine that $\sqrt[3]{2}$ is algebraic of degree 3 over $\mathbb{Q}$. (3)

6. Let R be the ring of all real-valued continuous functions defined on the closed unit interval, i.e., on $[0,1]$ and let $S = \{f \in R : f\left(\frac{1}{2}\right) = 0\}$. Justify whether S is an ideal of R or not. (3)

OR

What is radical extension? Conclude when a polynomial is called solvable by radicals? (3)

Group-C
(Long Answer Type Questions)   5 x 6=30

7. Justify that no group of order 40 is simple. (5)

8. Define group action and give an example. (5)

9. In the ring $(\mathbb{Z}_n, +, .)$ show that an element $\bar{m}$ is a unit if and only if the $\gcd(m, n)$ is 1. (5)

10. Evaluate $[\mathbb{Q}(\sqrt{3}, \sqrt{7}) : \mathbb{Q}(\sqrt{3})]$ and $[\mathbb{Q}(\sqrt{3}) : \mathbb{Q}]$. (5)

11. Let K be a field of characteristic 0. Let F/K be finite normal extension. Let $g(x) \in K[x]$ and E be the splitting field of $g(x)$ over F. Then deduce that E/K is normal extension. (5)

12. Justify that $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ is a normal extension of $\mathbb{Q}$. (5)

OR

Justify that the symmetric group S_3 has a trivial centre. (5)

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